Analytical Design and Simulation Rescaling of Magnetically Insulated Transmission Lines

Xiangmin Jin and Jiansheng Yuan

The State Key Lab of Power Systems, Department of Electrical Engineering
Tsinghua University, Beijing 100084, China

Abstract—Analytical solution and numerical simulation are two effective approaches for analyzing and designing the vacuum magnetically insulated transmission lines (MITLs) in multi-terawatts applications. It is introduced in this paper a scheme for designing a tapered-electrode-radius MITL used in the linear induction accelerator utilizing the laminar flow model to match self-limited anode currents of each segment. This preliminary design is validated with particle-in-cell numerical simulations. In order to enhance the power transportation efficiency the MITL has to be rescaled with the help of particle-in-cell simulations, so that the MITL operates in the load-limited equilibria with smaller amount of space electron flow. The design based on the numerical simulation is presented in this paper. However, the modification will cause more simulations and complicated manufactures.

DOI: 10.2529/PIERS060907211706

1. INTRODUCTION

In modern pulsed power systems due to the high power density, very high electric fields are produced at the metal surfaces in vacuum transmission lines. For example, a power density of 1 TW/cm$^2$ generates a 20 MV/cm electric field. Generally, when the local electric field on a cathode surface exceeds 200 kV/cm, electrons explosively emitted from the cathode and form a plasma sheath near the surface [1]. As a result of the radial electric field, electrons emitted from the cathode plasma sheath are pulled toward the anode and shunt the high voltage gap. Consequently, ordinary transmission lines have an energy flux limit of 100 MW/cm$^2$. However, the magnetic fields generated by the currents flowing in the system generate forces perpendicular to the electric fields to confine the electrons. When the currents become sufficiently large, the self-magnetic field can keep these electrons flowing parallel to the cathode and greatly reduce the loss of electrons to the anode. Thus, magnetic insulation is established. The electron trajectories and currents of magnetic insulation in a coaxial geometry are shown in Fig. 1.

![Figure 1: Schematics of currents and electron flow in an MITL.](image1)

![Figure 2: Schematic cross section of the voltage adder MITL.](image2)

2. NUMERICAL METHOD

During the past decades analytical models have been improved to give an understanding of the magnetically insulated transmission lines (MITLs) [2]. There are basically two types of analytical
equilibrium models, laminar flow and quasi-laminar flow models. Both models assume the energy and canonical angular are conserved. The difference between the two models is the character of the electron trajectories in the sheath. In laminar flow model the gap voltage is increased on a time scale long compared with the electron cyclotron period, thus the \( \frac{dE}{dt} \) drift moves electrons onto laminar flow orbits parallel to the plane of the gap. In quasi-laminar flow model the voltage in the gap is applied instantaneously and the resulting orbits are cycloid-like between the cathode and the edge of the electron sheath.

Besides, electromagnetic particle-in-cell (PIC) as used for the coupled simulations of charged particles and their electromagnetic fields is also a useful method for analyzing MITLs [3]. PIC utilizes a set of computational macro-particles for representing the electron clouds in the MITLs, and numerically solves the coupled self-consistent solutions of the micro-particle motion equations and the Maxwell equations in the simulation space. Large-scale PIC simulations are sometimes used for designing a whole large system, but these generally are fairly expensive, and suffer from long turnaround times [4]. However, PIC simulations are quite valuable for developing or modifying the results original obtained by analytical models. Therefore, we describe here the design scheme combining the analytical models and PIC simulations.

3. DESIGN OF MITL USING EQUILIBRIUM MODELS

The analytical equilibrium models have been proven to be useful in the design of cylindrical MITLs with no radii changes [5]. Generally in practice, MITLs have radial changes in the axial direction to fit for the loads dimensions. As shown in simulations and analysis, in an MITL with geometrical discontinuity, although a large amount of electrons inject from the upstream of geometries discontinuity, most of them are absorbed by the cathode downstream and have been replaced by emitted electrons with an increase in electron density and electron cloud boundary [6]. Therefore, the injected electrons from upstream act little influence on the downstream. As a result, equilibrium models can still be used in design and analysis of MITLs with radii geometrically changes.

Here we use a voltage adder MITL installed in the linear induction accelerator (shown in Fig. 2) as an example to describe the design process [7]. The voltage adder MITL sums the voltages of the pulses delivered from the voltage feeds. The voltages are added to the right so that all arrive simultaneously at the load \( Z_L \). The voltage adder has a structure with a constant outer anode radius and tapered inner cathode radii along the power-flow direction. Assume in order to drive a terminating load the adder MITL delivers a power with 750 kA and 20 MV. If the input voltages from the voltage feeds have the same peak value of 4 MV, so there will be 5 cavities on the voltage adder MITL. Besides, the outside radius of the anode cylinder is 38.1 cm which is defined by the system insulating stacks.

As we know that in an MITL that has no equilibrium electron loss, the anode current is constant along the length of the MITL. Consequently, operating in the efficient equilibrium the voltage adder MITL has a same anode current throughout every segment. Besides, assume that there is a final cylindrical MITL installed behind the voltage adder MITL which is sufficiently long to isolate the adder MITL from the terminating load, so when the MITL system operate in the equilibrium the anode current is determined by the self-limited current of the final MITL. Numerical simulations and measurements from many magnetically insulated devices have shown that the self-limited current tends to be near the minimum current on any voltage [8]. Accordingly, the anode current of the voltage adder MITL is the minimum current of the final MITL under its equilibrium voltage. The vacuum impedance \( Z_0 \) of each section of the voltage adder MITL can then be chosen so that the minimum anode current for each of the section voltage is the same. This choice allows matched self-limited flow for the local voltage and vacuum impedance all the way through the voltage adder MITL. Such an approach can only be used in the system which normally has a long final MITL; its current is determined by MITL itself but not the terminating load impedance. Determined by operation requests above the anode current all through each segment is 750 kA and the voltage adder MITL output voltage is 20 MV. Therefore, the local voltage of the five segments is respectively 4 MV, 8 MV, 12 MV, 16 MV and 20 MV. Because the voltage adder is negative-polarity (anode outside and cathode inside) and has a constant anode radius, the concept of matching self-limited current was applied to calculate the inner radii of each section of the voltage adder MITL.

With the initial parameters and laminar flow model equation for minimum anode current \( I_t \) [8]:

\[
I_t = I_0 g \gamma_t^3 \ln \left[ \gamma_t + (\gamma_t^2 - 1)^{1/2} \right] \tag{1}
\]
where the $I_0 = 2\pi m_0 c / \mu_0 e \approx 8500 \text{A}$, $m_0$ is the rest mass of the electron, $c$ is the velocity of light, $e$ is the charge of the electron and $\mu_0$ is the vacuum permeability. $g = 1 / \ln(r_a/r_c)$ is the cylindrical geometrical factor where $r_a$ and $r_c$ are the anode radius and cathode radius. $\gamma_l$ is the electron flow sheath boundary defined by

$$\gamma_l = 1 + eV / m_0 c^2$$  \hspace{1cm} (2)

where the $V$ is the voltage along the radial direction. Besides, $\gamma_l$ has the relation with anode voltage $\gamma_a$ as followed [8]:

$$\gamma_a = \gamma_l + (\gamma_l^2 - 1)^{3/2} \ln \left[ \gamma_l + (\gamma_l^2 - 1)^{1/2} \right]$$  \hspace{1cm} (3)

Hence for the given local voltage and required anode current for each segment, the corresponding cathode radii can be calculated using Eq. (1)$\sim$(3), so that in self-limited equilibria each segment has the same minimum anode current. The anode current curves for all segment voltages of the laminar flow model are given in Fig. 3 in which the abscissa represents the electron sheath edge voltage. The triangles on curve are the locus of the minimum current of all the segments. The solution for all the segments cathode radii of is shown in Table 1.

Table 1: Design results of laminar flow model and PIC simulation.

<table>
<thead>
<tr>
<th>MITL Segment</th>
<th>Segment Voltage (MV)</th>
<th>Model Cathode Radius (cm)</th>
<th>Rescaling Cathode Radius 1 (cm)</th>
<th>Rescaling Cathode Radius 2 (cm)</th>
<th>Rescaling Cathode Radius 3 (cm)</th>
<th>Actual Cathode Radius (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>4</td>
<td>33.8</td>
<td>27</td>
<td>29</td>
<td>29</td>
<td>33.7</td>
</tr>
<tr>
<td>D2</td>
<td>8</td>
<td>30.6</td>
<td>26</td>
<td>28</td>
<td>27.5</td>
<td>30.5</td>
</tr>
<tr>
<td>D3</td>
<td>12</td>
<td>27.8</td>
<td>25</td>
<td>26</td>
<td>26</td>
<td>27.6</td>
</tr>
<tr>
<td>D4</td>
<td>16</td>
<td>25.3</td>
<td>24</td>
<td>25</td>
<td>24.5</td>
<td>25.1</td>
</tr>
<tr>
<td>D5</td>
<td>20</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>22.7</td>
</tr>
</tbody>
</table>

The solution is not sensitive to the particular equilibrium flow model (laminar flow or quasi-laminar flow model) used in the calculation. Matching self-limited currents section to section produces the result that each section of the voltage adder MITL behaves like an infinitely long MITL with the local voltage.

4. PARTICLE-IN-CELL SIMULATIONS VALIDATION AND RESCALING

The voltage adder MITL designed above is now simulated by PIC simulations to exam the working features. In simulations the input voltage pulses applied at the five feeds are all chock waves with
10-ns rise-time and afterwards maintaining the voltage amplitude. The MITL output boundary condition is set to be a load-matched boundary which represents the isolation from the terminating load. Shown in Fig. 4 were the anode current and cathode current values at each segment, respectively from the laminar flow model and the simulation results at 35 ns into the pulse when a self-limited equilibrium has been established. As a result of matched self-limited design the simulation values of all of the segments are operating in the self-limited equilibrium with 750-kA anode currents. Whereas, from the simulations we can see that cathode currents at the 2nd to the 5th segment are lower than the analytical results. This is because at each geometrical discontinuity the disturbed electromagnetic field causes more electrons emit from the cathode and the electrons downstream tend to change their density to minimize the influence of the geometrical discontinuity [6]. While in the laminar flow model results are deduced by ignoring the effects of geometrical discontinuity of MITLs. Therefore, the cathode currents from the laminar model are higher than the simulations. Furthermore, in Fig. 4 the difference between the anode and cathode currents is the space electron flow which couples an amount of transportation power. From the results we can see that in the matched self-limited equilibrium more than half of the current is carried by the electron flow. If the load does not under-match the MITL, all these electrons and the currents associated with them will be lost before the load. Therefore, operating in the self-limited equilibrium the MITL has a limited transportation efficiency.

In order to enhance the transportation efficiency and lower the electron flow, dimensions of the MITL will have to be rescaled. As we know that the rate of the cathode to the anode current is higher when an MITL works in the load-limited equilibrium with the anode current locating on the super-insulated branches (left branches) of the curves in Fig. 3 and the electron flow sheath holding close to the cathode. Consequently, the MITL can be rescaled to make each segment operate under the load-limited equilibrium in order for high power transportation efficiencies. Since the last segment is connected with the final MITL, its cathode radius is still 23 cm as above. However the other segments can not be designed using the laminar flow method, because the free parameter in the model, the electron sheath boundary voltage, can only be determined beforehand in the self-limited equilibrium. Therefore, the cathode radii have to be calculated with the help of PIC simulations.

Based on analysis above we use PIC simulations to modify the cathode radii for the 1st to 4th segment of the adder MITL so as to adjust the anode current to be 750-kA at the local voltages. Three subsets of the modified dimensions are shown in Table 1. In order to decrease the manufacturing complexity each of the segment cathode radius is larger than the precedes by integral centimeters. It is shown in Figs. 5(a) and (b) that cathode currents and transportation efficiencies at each segment of these voltage adder MITLs compared with the analytically designed MITL. We can see that the cathode currents and transportation efficiencies of the rescaling MITLs are much higher than those of the analytically designed MITL. Therefore, one rule of rescaling MITLs for enhancing the transportation efficiency is that operating in the load-limited equilibria. Besides,
comparison of the three rescaling MITLs shows that the MITL with smaller step-downs in cathode radii has higher cathode currents and transportation efficiencies than the other two. And although the other two have the same starting and ending segments radii, the one with average cathode radius changes operates in a better condition than the other one. As known that at the geometrical discontinuity there are more electrons emitting from the cathode downstream to maintain the efficient impedance. Therefore the more abrupt the discontinuity is, the more electrons emit out so that the less the transportation efficiency will be. Accordingly, we get another designing rule to average and reduce the geometrical changes of MITLs.

However, with the changes in cathode radii the dimensions of the voltage feeds will have to be adjusted in order to match with the segments impedance and maintain the local voltages. Thereby, the simulation scales and manufacturing difficulties will be fairly large. Consequently, in actual project the radii of MITLs (also shown in Table 1) are commonly calculated based on analytical model results. The cathode current and transportation efficiency of the actual MITL are also shown in Fig. 6. Furthermore, it is used in actual projects the inductive rings (shown in Fig. 2) placed at the end of each feed which can help to reduce the electron flow by nearly 50%.

5. CONCLUSIONS

In practical applications, due to different geometrical features and operation requirements it is more practical combining analytical models and numerical simulations to optimize the design of MITLs. Based on the analysis and calculations conducted in this paper the design rules for MITLs can be summarized as follows.

1. The analytical models can be used in MITL designs even if the MITL has axial geometrical discontinuity, when a MITL is long enough compared with the input pulse duration. For a MITL with tapered radii segments, analytical model design methods of matched self-limited anode current for each of the MITL segment produces the result that each section operates in its self-limited equilibrium with the local voltage.

2. The power transportation efficiency of a MITL operating in the self-limited equilibrium is limited, because part of the power is coupled by the space electron flow that can not be delivered to the load. Consequently, the MITL will have to be rescaled with the help of PIC simulations to operate in the load-limited equilibrium in order to reduce the space electron flow and enhance the transportation efficiency.

3. In rescaling MITLs a good rule to follow is to average and reduce the geometrical changes in the radial direction.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (No. 50677028), SRFDP (No. 20050003007) and Tsinghua Basic Research Foundation.

REFERENCES