Entanglement Theory and a $q$ Analogue Entangled State

Huihui Zhang, A. Rauf, and Xiaoguang Zhou

Department of Engineering Optics, School of Information Science and Technology
Beijing Institute of Technology, Beijing 100081, China

Abstract—Entangled state and entanglement theory in the quantum information are presented. A $q$ analogue entangled state is first put forward in this paper. It exhibits a much richer structure than ordinary bipartite entanglement. Thus there will be abundant information in the quantum teleportation.

DOI: 10.2529/PIERS060906095324

1. INTRODUCTION

Quantum entanglement is not only one of the striking features in quantum mechanics but also a vital resource in some quantum information and quantum computation processes [1–4], such as teleportation, dense coding, quantum key distribution, and quantum computation. It was first noted by Einstein, Podolsky and Rosen (EPR) and Schrodinger. Entanglement describes a system composed of two or more particles, and exhibits the novel property that the result of measurement on one particle cannot be specified independently of the parameters of the measurements on other particles. Entanglement can exhibit the nature of a non-local correlation between quantum systems that have no classical interpretation. It has been noted that quantum teleportation can be viewed as an achievable experimental technique to quantitatively investigate quantum entanglement.

2. ENTANGLED STATE AND THE BELL BASIS

The entangled state can be expressed by the form:

$$|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B.$$  (1)

Bell basis is the maximum entangled state in the bipartite entanglement:

$$\begin{cases} |\phi^\pm\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle), \\ |\psi^\pm\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle). \end{cases}$$  (2)

The Bell basis measurement can become true by the simple quantum network as shown below:

Figure 1: Quantum network of identify Bell basis.
3. ENTANGLEMENT THEORY IN QUANTUM INFORMATION

Quantum information theory studies the transmission and processing of information when information itself is carried by quantum systems and is processed according to the laws of quantum mechanics [5–9]. The recent achievements in this field include the discovery of new ways of information transmission, of secure communications and the performance of some kinds of computation faster than with classical means. A key ingredient and fundamental resource in the development of all these tasks is quantum entanglement.

Quantum teleportation, proposed by Bennett et al., is the process that transmits an unknown qubit state from a sender (Alice) to a receiver (Bob) via a quantum channel with the help of sending some classical information. In original scheme, such a quantum channel has been represented by a Bell maximally entangled state or Einstein-Podolsky-Rosen (EPR) pair. Quantum teleportation has been considered for N-dimensional, and continuous variable states. Teleportation of the polarization photon state and coherent state of light field has been demonstrated in optical experiments. Other strategy in quantum information is related to use of a single-rail logic. So, teleportation of a qubit occupying only one mode via one-photon quantum channel was studied. Quantum teleportation of N-particle entangled state via N+1-particle quantum channel was considered in Experimental. Realization of teleportation through one-photon quantum channel has reported. And problems of quantum teleportation by employing Greenberg-Horne-Zeilinger (GHZ) entanglement have been studied. More general questions of quantum teleportation of two qubits involving noisy quantum channels are involved in. The standard method for encoding qubits in optics is to use the polarization degrees of freedom of single photon. However, it was recently recognised in that spatial encoding is easier, for example, to manipulate and construct universal quantum gates. First, address the issue of qubit transfer by the two-photon mode entangled quantum channel that takes simultaneously four modes. The source of the mode entanglement may consist of two non-collinear degenerated on frequency spontaneous parametric down converters with phase matching.

As pointed out by Bennett et al., in their original proposal for quantum teleportation, entanglement can be transferred through teleportation of two modes one of the particles forming the entangled state. This method, known as entanglement swapping, provides only partial teleportation of entanglement, an alternative method in which the entire mode entangled state is directly transferred from one place to another has been proposed. Use four-photon quantum channel constructed as a tensor product of two-photon mode entangled states to teleport the unknown mode entangled state. Quantum teleportation utilizing such four-photon quantum channel is not required special detectors distinguishing between one and two photon number states. The teleportation scheme of the entangled state through four-photon quantum channel may be more easily performed in practice than the teleportation schemes.

At present entanglement between two systems, i.e., bipartite entanglement, is quite well understood, but that between more than two systems, i.e., multipartite entanglement, remains still far from being satisfactorily known. In spite of that, multipartite entanglement has proven to play a superior role in recently emerging fields of quantum information processing and quantum computing since it exhibits a much richer structure than bipartite entanglement. Motivation for studying multipartite entanglement arises from many reasons some of which are listed now. First, multipartite entanglement provides a unique means to check the Einstein locality without invoking statistical arguments, contrary to the case of Bell inequalities using bipartite entanglement. Second, multipartite entanglement serves as a key ingredient for quantum computing to achieve an exponential speedup over classical computation. Third, multipartite entanglement is central to quantum error correction where it is used to encode states, to detect errors and, eventually, to allow fault-tolerant quantum computation. Fourth, multipartite entanglement helps to better characterize the critical behavior of different many-body quantum systems giving rise to a unified treatment of the quantum phase transitions. Fifth, multipartite entanglement is crucial also in condensed matter phenomena and might solve some unresolved problems Sixth, multipartite entanglement is recognized as a unreplaceable or efficient resource to perform tasks involving a large number of parties such as network teleportation, quantum cryptography, quantum secret sharing, remote entangling, quantum (tele)cloning, quantum Byzantine agreement, etc. Finally, multipartite entanglement is conjectured to yield a wealth of fascinating and unexplored physics. Current research in multipartite entanglement is progressing along two directions in parallel. One direction deals with problems such as how to classify, quantify, generate, control, distill and witness multipartite entanglement. The other direction proceeds to advance various applications exploiting the nonclassical multiway correlation inherent in multipartite entanglement. Many of the quantum information processing
Several quantum teleportation schemes of both discrete and continuous variables have been proposed. Continuous variable quantum teleportation of an unknown coherent states has been realized experimentally by employing a two-mode squeezed vacuum state as an entanglement resource. Recently, the teleportation schemes via the entangled coherent states have been discussed. The entanglement of entangled coherent states in vacuum environment by employing the entanglement of formation has been studied and find that the entanglement of formation of the entangled coherent states is sensitive with the relative phase when the amplitude is very small. A scheme of probabilistic teleportation via entangled coherent states, in which the amount of classical information sent by Alice is restricted to one bit. In this probabilistic teleportation scheme, a coherent superposition state can be probabilistically perfectly teleported via a properly chosen entangled coherent state. When the interaction with the vacuum environment is addressed, the mean fidelity of the scheme is studied and the optimal amplitude of the teleported state is found. Simultaneous distance-independent correlation between different systems called entanglement is the most characteristic trait that sharply distinguishes between quantum and classical worlds.

The study of quantum teleportation protocol is not only limited to qubits and qudits (systems in d-dimensional Hilbert spaces) but also to quantum systems in infinite dimensional Hilbert spaces. In real situations sender and receiver may not have shared maximally entangled state but some form of non-maximally entangled pure state (due to some imperfection at the source). Usually if one follows the standard protocol, one will not be able to complete the teleportation process with unit fidelity and unit probability. Rather, the fidelity will depend on the parameters of the unknown state and the teleportation will not be reliable. Of course, if one has several non-maximally entangled pairs, one can first perform entanglement concentration and then recover fewer perfect maximally entangled pairs, and then use one of them to teleport an unknown state using the standard protocol. If Alice and Bob have only one pair, they can perform local filtering first, and convert a non-maximally entangled pair to maximally entangled pair with certain probability. Then they can follow standard protocol. Teleporting an unknown state using any pure entangled state but using generalized measurements has been proposed. This has been termed as conclusive teleportation. Also, there has been a qubit assisted conclusive teleportation process. It has also been mentioned that teleportation with unit fidelity but less than unit probability is possible for a qubit encoded in a coherent state [10–13].

How to characterize and to measure the entanglement is a basic problem. Although many impressive progresses has been obtained during the past decade, there is no general qualitative and quantitative theory of entanglement. Among the known criterions for characterizing entanglement, entanglement witness (EW) is an important criterion for characterizing the presence of entanglement [14]. The EWs are Hermitian but not positive operators whose expectation value is positive in every separable state. The importance of EW’s stems from the fact that a given state is inseparable if and only if there exists an EW that detects it. Unlike the Peres-Horodecki criterion, which is a necessary and sufficient condition for determining entangled states on the low-dimensional quantum systems, for higher dimensions, this criterion is necessary one, the EW criterion is still sufficient and necessary one regardless of the dimension of quantum systems. In the other words, for any composite quantum systems the existence of the EW denotes the presence of entanglement. On the other hand, the EW, as operator, can be decomposed as the combination of the product projector operators, by which one can detect the entanglement experimentally. And the measurement can be implemented locally. Some authors introduced a kind of entanglement measure based on the optimal EW of states. For a given quantum state, how to construct its EW (if it has any) or how to prove it has no any EW is an interesting and hard problem. If one can find all EWs then the problem of characterizing entangled states is solved completely. Unfortunately, the determination of EWs for all states is also being computationally intractable. For some special states, however, their EWs can be constructed easily.

4. A $Q$-ANALOGUE ENTANGLED STATE

The $q$ deformation of Lie algebra (the so-called quantum group) has its origin in the quantum inverse problem and in physics it is related to the solutions of integrable systems, to particular problems of statistical physics and to a conformal field theory [15, 16]. Attention has been paid to the realization of the quantum group in terms of the $q$ analogue of the quantum harmonic oscillator ($q$-QHO). The quantum harmonic oscillator (QHO) is one of the most fundamental objects in quantum physics. In quantum optics the QHO describes, in particular, the single mode of the quantized cavity field.
On the other hand, up to now, there has not been an appropriate $q$ analogue to describe quantitatively the entanglement of two and more subsystems due to the high complexity of entanglement in multi-particle system. The purpose of this paper is to propose a new theory of entanglement using $q$ analogue of the quantum harmonic oscillator and give the basic form of entangled state.

Considering a two dimension $q$ analogue of the quantum harmonic oscillator, $x$ orientation is an ordinary harmonic oscillator (system A), the ground state is $|0\rangle$, the first excited state is $|1\rangle$, $y$ orientation is a $q$ analogue of the quantum harmonic oscillator (system B), the ground state is $|0\rangle_q$, the first excited state is $|1\rangle_q$. A form of entangled state can be proposed:

$$|\varphi\rangle = \cos \beta |0\rangle |1\rangle_q + \sin \beta |1\rangle |0\rangle_q$$

(3)

It is easy to prove that $|\varphi\rangle$ is entangled state by reduced density matrix and Schmidt law.

5. CONCLUSIONS

Quantum teleportation can be understood as a quantum computation and it has been suggested that quantum teleportation will play an important role as a primitive subroutine in quantum computation. Transmission of a rich state from one place to another is very important in the field of quantum information. One amazing discovery in this context is teleportation of a $q$ analogue entangled state. In this paper entangled state and entanglement in the quantum information is presented. A $q$ analogue entangled state is first put forward. Thus in the quantum teleportation we can have more abundant information by adjusting the value of $q$.

REFERENCES