Solving Electromagnetic Inverse Scattering Problems by SVRMs: a Case of Study Towards Georadar Applications

G. Angiulli, V. Barrile, and M. Cacciola

Università Mediterranea degli Studi di Reggio Calabria, DIMET
Via Graziella, loc. Feo di Vito-89100 Reggio Calabria, Italy

Abstract — In this paper, an heuristic approach based on Support Vector Regression Machines (SVRMs) is presented in order to solve a simple inverse scattering problem. Interesting results have been obtained, with a remarkable reduction of computational time. Future development of this work will interest the evaluation of the performances of SVRMs for detection of buried objects in stratified media. This is the starting point to develop models for typical Georadar applications.

DOI: 10.2529/PIERS060907144915

1. INTRODUCTION

Inverse electromagnetic scattering by objects that lie in free space or in layered media plays an increasing role in a wide range of technological applications. It is for this reason that, during the years, many methodological approach have been developed for a variety of problems involving e.g., one-dimensional and high dimensional unknowns in a homogeneous space with linear scattering approximations, and higher dimensional unknowns in a homogeneous space considering multiple scattering mechanism (see [1] and references within). Among the numerous technological applications of the electromagnetic inverse scattering, Ground-Penetrating Radar (GPR) also known as Georadar, is one of the most important. GPR is a near-surface remote sensing tool for detecting buried targets (see [2] and references within). Interesting applicative fields of GPR are measurements of object location into the subsoil (i.e., pipings, electric or telephonic cables, and so on) or soil characterization. In all these applications, it is very important to quickly obtain measures with an high level of precision in terms of location and dimensions of buried objects. However, the accurate modeling of a GPR is a complex task. Nevertheless, in order to obtain a quick GPR data processing, it is necessary to develop suitable models able to face the inverse problems.

In the last years, Soft Computing techniques, such as Neural Networks, Neuro Fuzzy Networks have been introduced in order to provide a fast treatment of the direct and inverse scattering problems (see [3, 4] and references within). Ability and adaptability to learn and generalize, fast real-time operation, and ease of implementation have made these techniques very popular. Very recently, another Soft Computing technique named as Support Vector Machines (SVMs), developed by Vapnik [5], has gained popularity due to many attractive features capable to overcome the limitations connected to Neural Networks. This is due to the Structural Risk Minimisation principle embodied by SVMs, which has been demonstrated to be more effective than the traditional Empirical Risk Minimisation principle employed by Neural Networks [5]. This different philosophy provides Support Vector Machines with a greater ability to generalise, if compared with Neural Networks. In this paper we investigate the performances of SVMs in the field of the inverse scattering. To this aim, our attention is focused on a simple electromagnetic inverse problem: the location of a perfect conducting thin metal strip immersed in the free space starting from the scattered field evaluated at a suitable number of measure points. This is a simple inverse problem, which can be solved exploiting the direct one (see [6, 7] and references within).

The paper is organized as follows: Section 2 gives the basics of SVRMs. Section 3 gives a brief account of the direct electromagnetic scattering problem exploited to collect data for SVRMs-based experimentations. Next, Section 4 describes the characteristics of collected dataset and hosts some discussions about retrieved preliminary results. Finally, Section 5 draws up our conclusions. All the computer codes exploited in this work have been implemented in Matlab®, using also a freeware toolbox for SVRMs [8].
2. A QUICK OVERVIEW OF SVRMs

SVRMs are learning machines that can be applied to regression problems (see [5] and references within). Their operation principle can be summarized as follows: like in standard regression problems, it is supposed that the relationship between the independent and dependent variables is given by a function $f$ plus the addition of some additive noise. The task is to find a functional form for $f$ which can correctly predict new cases that the SVRM has not been presented with before. Considering the problem of approximating the set of data $D = \{(x_1, y_1), . . . , (x_l, y_l)\}, x \in \mathbb{R}^n, y \in \mathbb{R}$ with a linear function $f(x) = \langle w, x \rangle + b$, the optimal regression function is given by the minimum of the functional

$$
\Phi(w, \xi) = \frac{1}{2} \|w\|^2 + C \sum_i (\xi^-_i + \xi^+_i)
$$

where $C$ is a user defined value, and $\xi^-_i, \xi^+_i$ are slack variables representing upper and lower constraints on the output of the system. If a linear regression is not possible due to non-linear relationships between data, SVRM non-linearly map the n-dimensional input space into a high dimensional space where a linear separation can be performed (see [9] and references within). In this case, a non-linear function, also known as kernel function (see [10] and references within), is used. This can be achieved by training the SVRM on a suitable training set. This process involves sequential optimization of an error (or loss) function (see [11] and references within). Depending on its loss function definition, two kinds of SVRMs can be recognized: $\varepsilon$-SVRM and $\nu$-SVRM. In this research, $\nu$-SVRM has been used (see [12] and references within).

3. CREATION OF THE TRAINING SET: DESCRIPTION OF THE DIRECT SCATTERING PROBLEM

In this section a brief account of the direct electromagnetic scattering problem, exploited to collect data for SVRM-based experiments, is given. In Fig. 1 is shown a perfect conducting two-dimensional metal strip, located in free-space within a box defined by coordinates $0 < x < x_{scan}$, $y_{scan} < y < 0$. $(x_c, y_c)$ are the coordinates of the strip centre. A number of $\kappa$ line filaments, working as either a source or a receiver, are located at an coordinate $y = y_Q$, in equally spaced points such that the $x$-coordinates of the first and last filament are 0 and $x_{scan}$, respectively. Each line filament illuminates the metal strip by a Tmz cylindrical electromagnetic wave. As well known, the incident fields induce electric current on the strip, which radiates a scattered Tmz wave, and this back-scattering field is detected by the same line filament in the receiving phase. Applied the boundary condition for the tangential electric field on the the strip, it is possible to write [13]:

$$
E_{z\text{scatt}}(x) = \frac{k_0\eta_0}{4} \int_{\frac{W}{2}}^{\frac{W}{2}} H_y^{(2)} (k_0|x - x'|) J_z(x') \, dx'
$$

Figure 1: Pictorial representation of the direct scattering problem discussed in Sec. 3. Please note an interesting $X_{scan}/W$ ratio, which is very important in GPR applications to detect and characterize small buried objects, according to the definition abilities of used Georadar apparatus.
where $E_{\text{scatt}}(x)$ is the tangential scattered electric field, $J_z(x')$ is the induced unknown current density flowing on the metal strip, $k_0$ is the free-space propagation constant, $\eta_0$ is the free-space intrinsic impedance, $H_0^{(2)}$ is the Hankel function of the second kind and zero order and $x$ and $x'$ represent the $x$-coordinates of the observation and source points, respectively. Solving Eq. (2) for $J_z(x')$ by MoM, the back-scattered field can be expressed as [13]:

$$E_{\text{scatt}}(\rho) = -\left(\frac{2}{\pi W}\right) \frac{k_0 \eta_0}{4} \int_{-\frac{W}{2}}^{\frac{W}{2}} H_0^{(2)}(k_0|\rho - \rho'|) J_z(x') \, dx'$$

(3)

where $\rho$ and $\rho'$ are the magnitude of vectors related to observation and source points respectively in a polar reference system.

4. DATA COLLECTION AND PRELIMINARY RESULTS

By using the described formulation and considering suitable numeric values for above described quantities, it is possible to solve the direct problem (2) by MoM and so collect a dataset. The values exploited in our numerical experimentation are reported in Table 1 (these data are the same of those employed in [7]). A number of 676 patterns have been collected in order to carry out training and test of a suitable SVRM (called MSLSVRM). It is possible to discriminate training and test elements by considering Fig. 2. In order to train MSLSVRM, it is necessary to adequately choose the kernel function. This operation has been carried out by fixing the other SVRM parameters, that is $C = 1$ and $\xi^-_i = \xi^+_i = \xi = 0.1 \quad \forall i$. In this way, an RBF kernel has been selected as the kernel having the best regression performances. Subsequently, in order to improve MSLSVRM
Table 1: Numerical quantities used to build the dataset.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f ) [MHz]</td>
<td>100</td>
</tr>
<tr>
<td>( W ) [m]</td>
<td>0.2</td>
</tr>
<tr>
<td>( x_{\text{scan}} ) [m]</td>
<td>5</td>
</tr>
<tr>
<td>( y_{\text{scan}} ) [m]</td>
<td>-1.5</td>
</tr>
<tr>
<td>( N )</td>
<td>5</td>
</tr>
<tr>
<td>( y_Q ) [m]</td>
<td>5</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2: Statistics about distances between observed metal strip’s centers and relative values estimated by MSLSVRM.

<table>
<thead>
<tr>
<th>Minimal error [m]</th>
<th>Maximal error [m]</th>
<th>Average error [m]</th>
<th>Standard deviation [m]</th>
<th>Root mean squared error (RMSE) [m]</th>
<th>RMSE percent ( \left( \frac{\text{RMSE}}{D} \times 100 \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>1.292</td>
<td>0.163</td>
<td>0.168</td>
<td>0.234</td>
<td>4.667%</td>
</tr>
</tbody>
</table>

performances, \( C \) and \( \xi \) values have been tuned to the following values: \( C' = 10 \) and \( \xi' = \xi \). Time elapsed to train MSLSVRM for estimation of \((x, y)\) coordinates of metal strips’ centres as a function of the backscattered field is 0.876 s. Test results are instantaneously communicated by MSLSVRM. Preliminary results obtained by our experimentation are very satisfying (Fig. 3), above all for the estimation of \( x \)-coordinates of metal strips’ center. The average distance between observed and simulated values of metal strips’ centres is equal to 0.163 m, the 3.25% on the greatest possible distance for the case of study, i.e., \( D = \| (0.11, 0) - (4.89, -1.5) \| \). For details on estimative errors, see Table 2.

5. CONCLUSIONS

In this paper a new Soft Computing approach for solving inverse scattering problems has been investigated. It is based on the use of heuristic SVRMs. In order to test the performances of this approach a simple case of study has been selected: the localization of a perfect conducting metal strip lying in a free-space environment. The obtained results shown that the proposed approach is accurate and fast, consequently very promising for real time applications. Future development of this work will interest the evaluation of the performances of SVRMs on real experimental data for detection of buried objects in stratified media. Therefore, presented method is a preliminary starting point to develop suitable models for typical Georadar applications.

REFERENCES