Coupling of Microstrip Lines Exciting the Magnetostatic Surface Waves

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Abstract—The coupling of microstrip lines which excite the magnetostatic surface waves is analyzed. The asymmetric excitation of microstrip line and the nonreciprocal propagation characteristic of the magnetostatic surface waves are considered. For the fast numerical calculation, the Jacobi polynomial is used as the basis function to satisfy the edge condition.

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1. INTRODUCTION

In order to use a magnetostatic surface wave (MSSW) in the delay line or for signal processing at the microwaves, one needs the characterization of the excitation methods. Microstrip line excitation of the MSSW in the multilayer structure including a ferrite film was calculated approximately by assuming that the current distribution is uniform in the cross-section and has no variation in the transmission direction [1]. Then, the full-wave spectral domain method analysis of the MSSW excited by the microstrip was shown to be numerically possible to account for the complex propagation constant along the transmission direction of the lossless microstrip line [2, 3]. It shows that the current distribution on the microstrip line is asymmetric in the cross-section and the propagation constant is complex which is understood as the power coupled to the radiation of the MSSW. In those analyses, surface current densities on the microstrip line may be expanded by Chebyshev polynomials multiplied by a factor that satisfies the microstrip line edge conditions asymptotically and is equal to the microstrip line with a dielectric substrate.

However, the size of the matrix to calculate the unknown coefficients of Chebyshev polynomials is large, for example, $11 \times 11$, in order to obtain the converging solution and so the computational time is great. In the microstrip line with a dielectric substrate, accurate solutions result even if an extremely small size matrix, for example, $2 \times 2$, is used. This is because qualitative natures such as the edge condition of the surface current distribution can be incorporated in the choice of basis functions. That is one of the features of the spectral domain method. Much better convergence can be obtained from a small size matrix if we choose a few basis functions which represent physical characteristics of the charge distributions on the strips. So we should choose basis functions representing the asymmetric current distribution on the microstrip line.

In this paper, the edge condition is derived using the Mexiner’s method and the Jacobi polynomial may be used as the basis function to satisfy the derived edge conditions. The effects of the nonreciprocal propagation of magnetostatic surface wave.

2. FULL-WAVE FORMULATION

The structure is shown in Fig. 1. A ferrite film is sandwiched between conductors covered with two dielectric layers, and an infinitely long microstrip excites this structure. Each layer is homogeneous and lossless. The magnetization direction of the ferrite coincides with the transmission direction of the microstrip and is taken as the $z$-axis and the cross-section is taken as the $x$-$y$ plane.

Since this structure is open and symmetric along the $z$ and $x$ axes, the field solutions may be assumed to have $\exp[-j(k_z z + k_x x)]$ dependences, where $k_z$ and $k_x$ are wave numbers to be determined along the $z$ and the $x$ axes, respectively. One may represent the field and the current as a superposition of these exponential eigenfunctions in the open domain, i.e., the Fourier integral with respect to $x$, as

$$ E(x, y, z) = e^{-jk_z z} \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(k_x; y)e^{-jk_x x} dk_x \tag{1} $$

where $E$ is the field and $\tilde{E}(k_x; y)$ is the modal amplitude corresponding to $k_x$. 

Taking their integrand from the Fourier integrals, one obtains
\[
\begin{bmatrix}
\tilde{E}_x(k_x) \\
\tilde{E}_z(k_x)
\end{bmatrix} = \begin{bmatrix}
\tilde{G}_{zz}(k_x, k_z) & \tilde{G}_{zx}(k_x, k_z) \\
\tilde{G}_{zr}(k_x, k_z) & \tilde{G}_{xx}(k_x, k_z)
\end{bmatrix} \begin{bmatrix}
\tilde{J}_z(k_x) \\
\tilde{J}_x(k_x)
\end{bmatrix}
\]
(2)
where \(\tilde{G}_{ij}(k_x, k_z)\), \(i = x, z\) and \(j = x, z\) are the Fourier transformed components of the dyadic Green’s functions, \(G_{ij}(x, z)\), with respect to \(x\) in the plane of \(y = h_1 + h_2\) and \(\exp(-jk_z z)\) dependences are suppressed.

Surface current densities in the microstrip may be expanded by polynomials satisfying the microstrip edge conditions for the fast numerical calculation. The singularities of the microstrip line edge on the ferrite substrate may be calculated using the Mexiner’s method [4, 5]. The calculated field behavior is
\[ J_z(x) \propto \rho^{-\xi} \]
(3)
where
\[ \xi = \frac{1}{2} \pm \frac{1}{2\pi} \ln \left[ \frac{(\mu - \kappa + 1)(\mu + \kappa)}{(\mu + \kappa + 1)(\mu - \kappa)} \right]. \]
(4)

From the consideration that the current distribution at the left edge is different from that at the right edge, the surface current distributions, \(J_i(x, z)\), \(i = z, x\), can be expanded in terms of the Jacobi polynomials, \(P_n^{(\zeta, \eta)}(x)\).

\[ J_i(x, z) = e^{-jk_z z} \sum_{n=0}^{N_i} C_{in} J_{in}(x) \]
(5)

\[ J_{zn}(x) = \begin{cases} (1 - \frac{x}{L})^\xi (1 + \frac{x}{L})^\eta P_n^{(\zeta, \eta)} \left( \frac{x}{L} \right) & |x| < L \\ 0 & |x| > L \end{cases} \]
(6)

Substituting the modal (Fourier) amplitudes of the surface current densities, \(\tilde{J}_{in}(k_x)\), with unknown coefficients \(C_{in}\) into Equation (2), one obtains two equations for \(\tilde{E}_i\), \(i = x, z\). One may satisfy the boundary condition in the microstrip by taking an inner product of \(\tilde{E}_i(k_x)\) and \(\tilde{J}_{in}(k_x)\) as
\[
\int_{-\infty}^{\infty} \tilde{J}_{im}^*(k_x) \tilde{E}(k_x) dk_x = \sum_j \sum_{m=0}^{N_j} C_{jn} \int_{-\infty}^{\infty} \tilde{J}_{im}^*(k_x) \tilde{G}_{ij}(k_x, k_z) \tilde{J}_{jn}(k_x) dk_x \]
(7)
where $j$ equals $x$ and $z$ and the asterisk denotes the complex conjugate.

Since the tangential electric fields are zero in the conducting microstrip and the surface current densities are zero outside the microstrip, the left-hand side of Equation (7) becomes zero everywhere in the plane $y = h_1 + h_2$ due to the Parseval's theorem. For nontrivial solutions of $C_{in}$, the values of $k_z$ may be obtained numerically by forcing the determinant of these simultaneous integral equations to be zero. With the values of $k_z$, $C_{in}$ may be obtained by solving the homogeneous simultaneous Equation (7), which gives the surface current densities in Equations (5)–(6).

For full-wave calculation, a microstrip excitation of the three-layer structure covered by conductors, shown in Fig. 1, is chosen with heights of layers, $h_1 = 250$ cm, $h_2 = 6.3$ µm, and $h_3 = 250$ µm, relative dielectric constants, $\varepsilon_1 = 1$ and $\varepsilon_2 = \varepsilon_3 = 10$, microstrip halfwidth, $L = 25$ µm, internal magnetic bias field for ferrite, $H_0 = 740$ Oe, saturation magnetization, $4\pi M_s = 1700$ Oe, and gyromagnetic ratio, $\gamma = 2.8$ MHz/Oe.

Numerical calculations of the surface current densities in the microstrip are found for the expansion of $J_x$ and $J_z$ by the Jacobi polynomials, respectively, to ensure numerical accuracy. Calculated current distributions across the microstrip ($J_z$ versus $x$) are asymmetric [3] and their asymmetry is reduced for the lower-frequency excitation.

From the calculated current distributions, one may obtain its modal amplitudes exciting the magnetic surface wave, as shown in Fig. 2, by taking the Fourier transform of the current distribution and picking the spatial frequency key corresponding to either $+x$ or $-x$ propagation for a given temporal frequency. Fig. 2 shows that the excitation amplitude for the $+x$ direction is larger than the $-x$ direction, while the uniform current distribution yields almost the same excitation amplitude values between the $+x$ and $-x$ directions. The difference in the excitation amplitudes between the $+x$ and $-x$ directions increases as the frequency increases.

Pointing vectors and their integration over the radiating surfaces give the radiating power and its direction carried by the magnetic surface waves. Fig. 3 shows that the magnitude of the power peaks at 3.86 GHz, its direction is $-112.5^\circ$ from the $z$-axis toward the $-x$ axis, and it is about 10 dB larger than that of the $+x$ direction. The directions of the real power flow are distributed in a sector about 30 degrees closer to the $x$ axis and, as the frequency increases, approach the $\pm x$ direction. The magnetostatic assumptions give the direction of power flow along the $x$ axis, which is different from the full-wave analysis.

3. CONCLUSION

The edge condition of a microstrip line having ferrite film in the frequency range where magnetostatic surface waves are excited is derived using the Mexiner’s method. It corresponds with the calculated variation of the surface current distribution. The Jacobi polynomial is chosen as the basis function to satisfy the obtained edge condition. Because the Jacobi polynomial may give the different conditions for each edge, that is an appropriate current basis function. Thus the current
basis function having the Jacobi polynomial may be expected to give fast convergence of numerical solution.

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REFERENCES