The Effect of Radiation Coupling in Higher Order Fiber Bragg Gratings

Li Yang¹, Wei-Ping Huang², and Xi-Jia Gu³

¹Department EEIS, University of Science and Technology of China, China
²Department ECE, McMaster University, Canada
³Department ECE, Ryerson University, Canada

Abstract—The effect of radiation coupling in second and higher order (marked as N-th order) fiber Bragg gratings is investigated by using coupled mode theory, where the radiation field is solved and expressed in terms of guided waves by using Green’s function. Different from the direct index coupling between two contra-propagating guided modes due to the N-th order diffraction, the radiation coupling lead to the indirect self or mutual complex coupling between the two guided waves via the diffractions lower than N-th order. It will become dominant and result in some interesting properties potentially useful in the DFB fiber laser at some special cases such as a rectangular index modification with the duty cycle equals 0.5. For this reason, the dependences of coupling coefficients, spectrum characteristics and the bandwidth of Bragg reflection peak on the parameters of gratings with rectangular profiles, such as duty cycle, index change and grating length, are simulated and discussed in this paper.

DOI: 10.2529/PIERS061006135306

1. INTRODUCTION

Over the past decades, much theoretical and experimental work has been devoted to the first order fiber Bragg Grating (FBG), while there are fewer reports on the higher order FBGs possibly because of the limitation of the fabrication techniques. However, the distinct properties of higher order FBGs indicate potential applications in laser, sensor and etc, which draw more and more interests [1–5].

One notable property in higher order FBGs is the complex radiation coupling via the diffraction lower than N-th order which is the order of FBG. It means, in the total coupling coefficient, in addition to index coupling coefficient due to the direct coupling between two contra-propagating guided waves similar to the first order gratings, there is radiation coupling coefficient indicating the indirect coupling between the two guided waves via the excitation and resonant of the radiation wave. Since the index coupling coefficient is proportional to the index change if it does not equal zero, while radiation coupling coefficient is proportional to the square of the index change, the effect of radiation coupling will be prominent only when the former one is around zero at some special cases such as a rectangular index modification with the duty cycle equaling 0.5 [3]. At such cases, the complex radiation coupling coefficient provides possibility to control the total coupling coefficient and then the spectrum characteristics in a wider range, which has been proved to be useful in DFB laser based on multilayer planar waveguides [6, 7].

On the other hand, in experiments, usually, the perturbed index profile in FBG is almost sinusoidal; and it will be more similar to rising-cosine after the saturation of the index modification [5]. During the growth of the index change, the higher order gratings attributed to the effect of the higher order Fourier expansion coefficients show up; meanwhile, the radiation coupling enhances gradually, which offers the possibility and necessity to study the effect of radiation coupling in higher order FBGs.

In this paper, the spectrum characteristics of higher order FBGs with rectangular index modification in a theoretical model are investigated by using coupled mode theory. Hereinto, the radiation fields are determined in terms of guided waves by using Green’s function method. The influence of grating parameters such as duty cycle, index change and grating length on the coupling coefficients and the bandwidth of Bragg reflection peak are simulated and discussed.
2. THEORETICAL MODEL AND BASIC FORMULATIONS

A schematic diagram of a cylindrical symmetric single mode FBG in the theoretical model is shown in Fig. 1. In cylindrical coordinates based on the \(\exp(j\omega t)\) time variation, the refractive index in the perturbed optical fiber can be expressed as,

\[
\tilde{n}^2(r, z) = \begin{cases} 
\tilde{n}_{\text{co}}^2 & 0 < r \leq r_0 \\
\tilde{n}_{\text{cl}}^2 & r > r_0 
\end{cases} + \sum_{m=-\infty}^{\infty} \eta_m \exp(-j\frac{2m\pi}{\Lambda}z) + \sum_{m=-\infty}^{\infty} \eta_m \exp(-j\frac{2m\pi}{\Lambda}z),
\]

where \(r_0, \tilde{n}_{\text{co}}\) and \(\tilde{n}_{\text{cl}}\) are respectively the core radius, the refractive index of the core and the cladding in the unperturbed fiber, while \(n_0\) is the refractive index of the core in the reference fiber. The Fourier expansion coefficients of the periodic index modification is

\[
\eta_m = \frac{2n_{\text{co}}\delta n}{\Lambda} \int_{-\Lambda/2}^{\Lambda/2} f(z) \exp\left(j\frac{2m\pi}{\Lambda}z\right) dz
\]

where \(f(z)\) is the profile of the periodic modification.

Following the previous analysis for a \(N\)-th order grating, the electrical field is expressed in terms of two contra-propagation fundamental guided waves (named as \(\text{LP}_{01}\) mode in fiber) and radiation waves due to the resonant via the diffraction lower than \(N\)-th [3, 6, 7]. For a fiber with a small index difference, the radiation waves \(E_{r0}^{\text{rad}}\) can be solved and expressed in terms of the two guided waves by Green’s function in a uniform region [3]. Then from the partial differential equation the electrical field of \(\text{LP}_{01}\) mode satisfies, the following coupled mode equations are obtained as follows,

\[
\begin{align*}
\frac{da(z)}{dz} &= -j (\delta + \kappa_{\text{sa}}^\text{rad}) a(z) - j \left(\kappa_N + \kappa_{\text{m}(ab)}^\text{rad}\right) b(z) \\
\frac{db(z)}{dz} &= j (\delta + \kappa_{\text{sb}}^\text{rad}) b(z) + j \left(\kappa_N - \kappa_{\text{m}(ba)}^\text{rad}\right) a(z)
\end{align*}
\]

In (3), \(\delta\) is the detuning factor,

\[
\delta = \beta - \beta_0 = \beta - \frac{N\pi}{\Lambda}
\]

(4)

\(\kappa_{\pm N}\) is the index coupling coefficient analytically expressed as

\[
\kappa_{\pm N} = \frac{\pi Y A_0^2 \tilde{A}_{00}^2 \eta_+ \eta_-}{4\beta_0 J_0^2(k_0 r_0)} \left[J_0^2(k_1 r_0) + J_1^2(k_1 r_0)\right] = \frac{k_0}{2} \eta_{\pm N} \Gamma
\]

where \(Y\) and \(A_0\) are respectively the mode admittance and normalized amplitude of \(\text{LP}_{01}\) mode in the reference fiber. \(J_0(k_1 r_0)\) and \(J_1(k_1 r_0)\) are Bessel Functions, where \(k_1\) is the transverse propagation constant of \(\text{LP}_{01}\) mode in the core. More briefly, \(\kappa_{\pm N}\) is expressed as the product of the half of the propagation constant in free space, \(k_0\), the \(\pm N\)-th order Fourier expansion coefficient, \(\eta_{\pm N}\), and the confinement factor of \(\text{LP}_{01}\) mode, \(\Gamma\).

When \(N > 1\), the analytical expression of self radiation coupling coefficients \(\kappa_{\text{sa}}^\text{rad}\) and \(\kappa_{\text{sb}}^\text{rad}\), mutual radiation coupling coefficients \(\kappa_{\text{m}(ab)}^\text{rad}\) and \(\kappa_{\text{m}(ba)}^\text{rad}\) are obtained by using Green’s function method and analytically expressed as follows,

\[
\begin{align*}
\kappa_{\text{sa}}^\text{rad} &= \sum_{n=1}^{N-1} \kappa_{n, -n}^\text{rad} \\
\kappa_{\text{sb}}^\text{rad} &= \sum_{n=1}^{N-1} \kappa_{n, -n}^\text{rad} \\
\kappa_{\text{m}(ab)}^\text{rad} &= \sum_{n=1}^{N-1} \kappa_{n, -n}^\text{rad} \\
\kappa_{\text{m}(ba)}^\text{rad} &= \sum_{n=1}^{N-1} \kappa_{n, -n}^\text{rad}
\end{align*}
\]

(6)

\[
\kappa_{n, j}^\text{rad} = -j \frac{\pi Y A_0^2 \tilde{A}_{00}^2 \eta_+ \eta_-}{4\beta_0 J_0^2(k_0 r_0)} \left(k^2 - k_{11}^2\right) \int_{0}^{r_0} C(r) J_0(k_1 r) r dr, \quad n = 1, 2
\]

(7)

\[
C(r) = k r J_0(k_1 r) \left[H_0^{(2)}(kr) J_1(kr) - H_1^{(2)}(kr) J_0(kr)\right]
\]

\[
+ r_0 \left[k H_1^{(2)}(kr_0) J_0(k_1 r_0) - k_1 H_0^{(2)}(kr_0) J_1(k_1 r_0)\right] J_0(kr)
\]

(8)
where $k$ is propagation constant in uniform region. For symmetrical structure, there are $\kappa_N = \kappa_{-N}$ and $\kappa_{\text{rad}} = \kappa_{\text{rad}}^a = \kappa_{\text{rad}}^b = \kappa_{\text{rad}}^{(ab)} = \kappa_{\text{rad}}^{(ba)}$. The total coupling coefficient is defined as $\kappa_{\text{total}} = \kappa_N + \kappa_{\text{rad}}$.

The reflection, transmission spectrums are obtained by solving the coupled-mode equations via transfer matrix method. The radiation power is obtained by subtracting the reflection and transmission power from the total power.

3. NUMERICAL RESULTS

The effect of radiation coupling in second order FBG with theoretically rectangular index modification is concentrated on. The designed Bragg wavelength is set at 1.55 $\mu$m.

The effect of duty cycle is firstly studied. Seen from Fig. 2, the index coupling coefficient sinusoidally varies, obtaining the maximum values when duty cycle equals 0.25 and 0.75, equaling zero and changing sign when duty cycle equals 0.5. The radiation coupling coefficient varies with the square of sinusoidal function, obtaining the maximum value when duty cycle equals 0.5. These are all determined by the corresponding analytical expressions of coupling coefficients and the Fourier expression properties of the periodic rectangular function. It is thus not difficult to understand the radiation wave becomes prominent with the decrease of the reflection power only when the duty cycle is about 0.5, seen from Fig. 3. The prominent occurrence of the radiation wave corresponds to the narrowest 3 dB bandwidth of the Bragg reflection peak seen from Fig. 4. It also leads to a blue shift of the Bragg wavelength, seen from Fig. 5. Conclusively, at the special case of duty cycle around 0.5, the dominant complex radiation coupling results in a decrease of the total coupling coefficient and offer a possibility to control the total coupling coefficient in a wider range, which thus leads to some properties different from the first order FBG, such as a lower reflection power, a narrower and blue-shifted Bragg reflection peak.

Moreover, taking the advantage of other grating parameters, the effect of radiation coupling mentioned above can be further controlled. Seen from Figs. 6–8, due the enhancement of radiation coupling with the increase of the index modification, the reflection power will increase and the Bragg reflection peak will be broader. On the other hand, the increase of the grating length will
result in the increase of the reflection power and the decrease of the Bragg reflection bandwidth, as shown in Figs. 9–10.

4. CONCLUSION

By using Coupled mode theory and Green’s function method, the effect of radiation coupling in higher order FBG is modeled and simulated. For a second order FBG with rectangular index modification, when the duty cycle is around 0.5, the radiation wave becomes prominent. Meanwhile, with the elimination of the usually dominant index coupling, the complex radiation coupling is dominant in the total coupling coefficient. It results in the decrease and a wider controllable range for the total coupling coefficient, and then the decrease of reflection power and Bragg reflection bandwidth. These properties are simultaneously influenced by the index change and grating length. The study will be useful for the applications of high order FBGs, such as the design of DFB fiber laser.
ACKNOWLEDGMENT
Thank the Chinese Academy of Sciences for providing Li YANG with the fellowship and the Ontario Photonics Consortium for financial support.

REFERENCES