Numerical Simulation for the Effective Conductivity of Composite Medium in High Frequency

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Abstract — Because of the disunity of resolution and exploration range in applied geophysics, the effective medium theory (EMT) should be developed to help us to understand the geological microstructure. We extended the EMT to complex permittivity in high frequency, and calculated the imaginary part of effective complex permittivity of composite as the effective conductivity using Finite-Difference Time-Domain (FDTD) numerical method. The result we obtained is the intrinsic property of the equivalent medium, which has explicit geological signification to satisfy the requirement of interpretation.

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1. INTRODUCTION

The electromagnetic (EM) method in applied geophysics can map the distribution and property of geological object underground efficiently, but there are always some important parameters referring to microstructure we cannot detect directly. For example, petroleum geologists have strong interest in the porosity and saturation of oil-bearing rocks. Unfortunately, most geophysical methods do not have enough resolution to study microscale anomaly, so balance between the exploring range and investigating resolution become a problem.

Effective medium theory (EMT) is a good solution to simplify the composite problem to equivalent homogeneous medium [3]. So researchers can concentrate on the relationship between heterogeneity in microscale and effect in macroscale. Maxwell, Bruggeman, Rayleigh, Pauly and Schwan all contributed their distinguished works on how to predict the effective electric properties of composite exactly. Archie formula is a famous empirical mixing law in geophysics, by which the stratum physical parameters can be inferred from resistivity data after inversion.

However, most researches paid attention to static or low frequency field (under 1 MHz) considering conductivity or permittivity singly, and achievements of microwave mainly came from sample test in laboratory. The propagation of HF EM wave is governed by Helmholtz Equation ($\vec{H}$ field has the same format)

$$\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0, \tag{1}
$$

where $\gamma = j\omega\sqrt{\mu\varepsilon}$ is propagating constant, and complex permittivity $\varepsilon = \varepsilon - j\frac{\sigma}{\omega}$ containing both permittivity and conductivity characters the intrinsic electric properties of medium. Therefore, the EMT of complex permittivity is needed.

But the complex permittivity is a frequency-dependent parameter without any geological information; it also cannot satisfy the customary requirement of conductivity mapping by geophysicists. We noticed that signal attenuation (skin depth) due to conductive medium is the main effect during wave propagation, so it is rational to substitute the multi-phase mixture with homogeneous medium having equivalent attenuation ability. There are two main factors causing EM energy decay in earth medium: conductive loss and polarization loss. In our research, to meet the need of geological interpretation, we aimed at calculating the imaginary part (multiplied by circular frequency) of effective complex permittivity of the composite, which is defined as the effective conductivity in HF. This effective conductivity $\sigma_{\text{eff}}$ is the intrinsic property of hypothetical equivalent medium following Ohm’s Law

$$J = \sigma_{\text{eff}} \vec{E} \tag{2}$$

and related only with conductive current by the transmission of electrons or ions in rocks.

In this paper, we adopt “black box” method applying numerical simulation to infer the medium properties from surveyed EM field behavior (as output). Our approach avoided some impractical preconditions in analytical modeling before, and would not be disturbed by irrelevant factors as in sample test. We can also simulate the extreme case using difference gridding while Maxwell Equation is still obeyed.
2. MODELING

To be easily-computed and comparable, our research began with a simplified two-dimensional mixing system abstracted from real-world rock background. We suppose that:

(1) There are two phases in the system, namely matrix and impurity; the grains of impurity are embedded in the matrix randomly.

Although high frequency EM wave has very short wavelength, the geological heterogeneousness is always much tinier than the exploring wavelength. For example, pore size in coarse grain sandstone of Woodbine C, Kurten Oilfield, Southeastern Texas, is scaled in hundreds \(\mu m\) [4], and if we use EM wave at frequency 1 GHz, the wavelength will be several decimeters (general conductive ground) at least. It is Rayleigh region according to radar theory. Because of such striking scaling contrast, we do not need to care about the shape of impurity grain. So the simplest spherical scatter is chosen to describe the heterogeneousness in rocks. Figure 1 shows grid simulation of random distribution in disseminated sandstone. Also, the grain size and sorting grade are adjustable.

![Grid Simulation](image)

Figure 1: Analog of two-phase mixture (The white spots are embedded scatters; the matrix medium is black; the dotted area is about 10 cm\(^2\) divided into 340 × 300 cells).

(2) What we simulated is a differential area \(dS\) of 2D EM field in the earth. It is small enough to not take the spatial asymmetry of EM wave into account; therefore, the hypothesis of incident plane wave can give good approximation. Farther, we think that the current density \(J\) and electric intensity \(E\) are not strongly-varied in such a small area, so the effective conductivity of \(dS\) can be obtained by average Ohm’s Law [5]

\[
\sigma_{\text{eff}} = \frac{\langle J \rangle}{\langle E \rangle},
\]

(3) The intrinsic properties of both materials in our system, including conductivity, dielectric constant, magnetic permeability, magnetic conductivity, are evaluated beforehand. If they are frequency-dependent or temperature/pressure-dependent, supposedly, there have been precorrection before computation. The geochemical interaction is also not considered. It means that we only want to know how dose the media distribution and electric properties affect the effective conduc-

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<table>
<thead>
<tr>
<th>Parameters (Unit)</th>
<th>Typical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductivity (S/m)</td>
<td>1, 10(^{-1}), 10(^{-2}), 10(^{-3}), 10(^{-4})</td>
</tr>
<tr>
<td>Relative Dielectric Constant</td>
<td>1, 5, 20, 80</td>
</tr>
</tbody>
</table>
tivity. Some typical parameters were determined by near-surface petrophysics as shown in Table 1 [6].

3. NUMERICAL METHOD

Finite-Difference Time-Domain (FDTD) is a well-developed finite difference method operating in time domain, by which we can obtain the numerical solution of Maxwell equation with good speed and enough accuracy. And we feel very free to put arbitrarily-shaped impurity into the difference grid.

In this research, we used centered difference with two-order accuracy and perfectly matched layer (PML) absorbing boundary condition (Berenger, 1994). The wave was 2D TM mode concerning three components \(\vec{E}_z, \vec{H}_x, \text{ and } \vec{H}_y\). The time series of sinusoidal \(\vec{E}_z\) source was generated by 1D FDTD, and added into the impurity area via connective boundary surrounding the white-dotted rectangle in Figure 1. Totally, there were three boundaries from outside to inside: absorbing boundary, connective boundary and target boundary.

The grid size \(dx\) (or \(dy\)) in our program is constantly \(10^{-4}\) m to give enough fitting to the grain size in rocks, especially sedimentary rocks. We designed an even gridding system of 300×300 size following staggered cell style (Yee, 1966). The impurity distribution was realized by designating different electric parameters at the gridding nods randomly. According to Courant’s condition and general geological background, we conservatively chose \(dt = 1.67 \times 10^{-13}\) s as time separation between two iterations. The iteration would not stop until the field intensity in each nod become stable. Repeated computations and averaging is helpful to improve the final accuracy.

All computation has done on the platform of Mathworks MatLab 7.0, by which we were able to monitor the frame movie of wave propagation while running.

4. VALIDITY TEST

We tested a set of data from real world, and compare the computational result with traditional formulaic solution to verify the validity of our numerical method.

The Tengger Desert in Northeastern China is famed for silver sand having very small grain diameter. A sand-air mixing sample from there was measured in wave-guide at frequency 33.5 GHz. The lab investigation showed the complex relative permittivity of that sample was \(5.43 - j0.074\), and volume fraction of air was 0.08 [9].

Because of the EM wavelength forty times greater than the pore size in the sample, it is reasonable to refer this composite problem in dynamic EM field to Bruggeman’s formula [10]

\[
f_1 \frac{\varepsilon_1 - \varepsilon_{\text{eff}}}{\varepsilon_1 + 2\varepsilon_{\text{eff}}} + f_2 \frac{\varepsilon_2 - \varepsilon_{\text{eff}}}{\varepsilon_2 + 2\varepsilon_{\text{eff}}} = 0
\]

where \(f_1, f_2, \varepsilon_1, \varepsilon_2\) are volume fractions and complex permittivities of matrix and impurity respectively, and \(\varepsilon_{\text{eff}}\) is the effective complex permittivity. After substituting variables: \(f_1 = 0.92, f_2 = 0.08, \varepsilon_1 = 5.43 - j0.074, \varepsilon_2 = 1\) (pure air) into Bruggeman’s formula, we obtained \(4.947 - j0.0654\) as the effective complex permittivity of sand-air composite, so the effective conductivity of analytical solution was 0.122 S/m.

We also ran our program with the same parameters above, and then got a group of simulating results by five repeating computations as listed in Table 2. Statistic analysis demonstrated that the RMS was in allowable tolerance; therefore the arithmetical average value was accepted. The numerical solution showed good agreement to analytical solution within the error of 0.24%.

<table>
<thead>
<tr>
<th>Simu. 1</th>
<th>Simu. 2</th>
<th>Simu. 3</th>
<th>Simu. 4</th>
<th>Simu. 5</th>
<th>Average</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.126</td>
<td>0.121</td>
<td>0.123</td>
<td>0.125</td>
<td>0.123</td>
<td>0.124</td>
<td>0.17%</td>
</tr>
</tbody>
</table>

More numerical tests have proved that the initial amplitude of incident wave and gridding number did not impact the final result significantly.
5. COMPUTATIONAL RESULTS

Several groups of results from numerical experiments are summarized in Table 3.

Table 3: Simulation results.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>10^6 Hz</td>
<td>0.0001 S/m</td>
<td>1</td>
<td>0.001 S/m</td>
<td>1</td>
<td>80/20</td>
<td>0.00028 S/m</td>
</tr>
<tr>
<td>10^8 Hz</td>
<td>0.0001 S/m</td>
<td>1</td>
<td>1 S/m</td>
<td>1</td>
<td>20/80</td>
<td>0.7968 S/m</td>
</tr>
<tr>
<td>10^9 Hz</td>
<td>0.0001 S/m</td>
<td>1</td>
<td>0.001 S/m</td>
<td>1</td>
<td>60/40</td>
<td>0.00046 S/m</td>
</tr>
<tr>
<td>10^9 Hz</td>
<td>0.0596 S/m</td>
<td>5.43</td>
<td>0</td>
<td>1</td>
<td>90/10</td>
<td>0.0536 S/m</td>
</tr>
<tr>
<td>10^9 Hz</td>
<td>0.01 S/m</td>
<td>5.0</td>
<td>1 S/m</td>
<td>80</td>
<td>85/15</td>
<td>0.1679 S/m</td>
</tr>
<tr>
<td>10^9 Hz</td>
<td>0.0596 S/m</td>
<td>5.43</td>
<td>0</td>
<td>1</td>
<td>40/60</td>
<td>0.0238 S/m</td>
</tr>
<tr>
<td>10^9 Hz</td>
<td>0.1 S/m</td>
<td>1</td>
<td>1 S/m</td>
<td>1</td>
<td>80/20</td>
<td>0.2804 S/m</td>
</tr>
<tr>
<td>10^10 Hz</td>
<td>0.0001 S/m</td>
<td>1</td>
<td>1 S/m</td>
<td>1</td>
<td>60/40</td>
<td>0.3956 S/m</td>
</tr>
<tr>
<td>10^10 Hz</td>
<td>0.0001 S/m</td>
<td>1</td>
<td>1 S/m</td>
<td>10</td>
<td>60/40</td>
<td>0.4010 S/m</td>
</tr>
</tbody>
</table>

6. DISCUSSION & CONCLUSION

(1) The EMT can be studied by FDTD methods, but the numerical simulation is too time-consuming. We have to suffer much more iterations if the wavelength is much greater than the heterogeneousness size. So we are looking forward to the advance of computational electromagnetics.

(2) Although our program agreed Bruggeman’s formula well, we had better conclude after all band verification in further work. If possible, laboratory experiment or real rock test must be carried out.

(3) In conductivity contrasted two-phase medium, there will be a “dominant” phase having much greater conductivity relatively. The effective conductivity is controlled by the conductivity and volume fraction of the dominant phase.

(4) If the conductivities of two phases are close, the effective conductivity is contributed by both phases with their volume fractions as weighting. In this case, the difference of permittivity should be seriously concerned, if any.

(5) The grain size of impurity doesn’t affect effective conductivity apparently, but we have observed slight decrease of effective conductivity with the grain growth. Absolutely, the simulating accuracy becomes worse when the grain size is larger.

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REFERENCES

