A 3D-2D AGILD EM Modeling and Inversion Imaging

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Abstract — In this paper, we propose a 3D-2D Advanced Global Integral and Local Differential (AGILD) Electromagnetic (EM) modeling and inversion imaging algorithm in the cylindrical coordinate system. Transmitting sources are excited in the center hole and the EM field datas are received in many surrounded receiver holes. We use the 3D AGILD EM inversion and the above data configuration to make the multiple holes’ image. Our algorithm is called as the 3D-2D AGILD EM modeling and inversion imaging. This algorithm effectively reduces the computational cost of the full 3D nonlinear inversion and increases the resolution of the 2D inversion. The multiple 2D cross holes’ inversion iteration are developed. 3D AGILD EM modeling method reduces matrix solving cost and eliminates the error reflection on the artificial boundary. The 3D-2D AGILD EM inversion reduces the 3D inversion ill posed and the 3D full matrix solving cost. Many synthetic data inversion tests show that the 3D-2D multiple cross holes inversion can obtain high resolution image. Our 3D-2D AGILD EM modeling and inversion imaging method will be useful for oil exploration, earthquake exploration, geophysical engineering, environment characteristic monitoring, nondestructive testing, medical imaging, and material and nanometer materials sciences and engineering.

DOI: 10.2529/PIERS060907233117

1. INTRODUCTION

There have been several research papers on the electromagnetic modeling and inversion and its application in the geophysics and other scientific and engineering fields. Habashy et al. proposed ‘Beyond the Born and Rytov Approximation’ in 1993 [1]. Habashy and Oristaglio proposed ‘Simultaneous Nonlinear Reconstruction of Two-dimension Permittivity and Conductivity’ in 1994 [2]. ‘Some Uses (and abuses) of Reciprocity in Wave Field Inversion’ was also proposed by Oristaglio and Habashy in 1996 [3]. Hohmann proposed ‘Three-dimensional Induced Polarization and Electromagnetic Modeling’ in 1975 [4]. We have proposed the Global Integral and Local Differential (GILD) EM modeling and inversion method since 1997. These papers have been published in the Geophysics [5], Physical D [6], SEG book, and PIERS conference proceedings [7–10]. In [5], we detailedly described the GILD EM modeling and inversion method. We proposed the advanced GILD modeling and inversion in PIERS 2005 in Hangzhou [7] and PIERS 2006 in Cambridge [8]. These methods are sorted as AGILD methods. The Stochastic AGILD algorithm is called as SAGILD EM modeling and inversion [9, 10]. According to the AGILD EM modeling and inversion, the 3D-2D AGILD EM modeling and inversion method for the imaging of multiple cross holes is proposed in this paper.

We consider the data site configuration in which the sources are in the center hole and the receivers are in the surrounded holes, as shown in Figure 1. This data site configuration is useful for oil geophysical exploration, environment engineering, and mine exploration. Similar data site is available for the medical detection, material study and nondestructive testing. When a laser source is excited in the center and micro and nanometer probes are in the surrounded sensors, we can construct a nano-device to study nanometer materials. We call this data configuration as Center Source and Multiple Receivers in the Surrounded site (CSMRS).

The general 3D EM inversion has high computational cost for the CSMRS configuration and the 2D and 2.5D EM inversion approaches can not interpret the spatial abnormality of 3D parameters in the rotational direction. In this paper the 3D-2D AGILD EM modeling and inversion imaging method can greatly reduce the computational cost, eliminate the error reflection on the artificial boundary, and obtain reasonable resolution. Using cylindrical coordinate system, the strip magnetic field differential integral equation in the boundary of cylindrical strip and center hole sub domain, and the Galerkin magnetic field equation in the internal cylindrical domain are coupled to construct 3D AGILD EM modeling. By restricting the 3D modeling data and Green’s function into 2D
cylindrical sectors, the 3D into 2D restricting strip magnetic field differential integral equation in the strip boundary cylindrical sector and the 3D into 2D restricting Galerkin magnetic field equation in the internal cylindrical sector are coupled to construct the 3D-2D AGILD EM inversion. Three adjacent 2D section AGILD EM inversions are coupled to construct the 3D-2D AGILD EM inversion iteration.

We use the 3D-2D AGILD EM inversion imaging method to make multiple holes’ images. The synthetic data and field data interpreting tests show that the 3D-2D AGILD EM inversion has reasonable resolution of 3D parameter in the radial, vertical and rotational direction and reduces the computational cost.

The arrangement of this paper is as follows. Introduction has been described in Section 1. The 3D AGILD EM modeling in cylindrical coordinate is described in Section 2. In Section 3, we describe the 3D AGILD EM inversion. The 3D-2D AGILD multiple cross holes’ inversion and their multiple cross holes data configuration are described in Section 4. The 3D-2D AGILD multiple cross hole images are presented in Section 5. In Section 6, we describe the conclusion.

2. 3D AGILD EM FIELD MODELING

2.1. The 3D Strip Magnetic Field Differential Integral Equation

By substituting the field and coordinate transformation between the rectangle and cylindrical coordinate system, we derive the 3D strip magnetic field differential integral equation [7] in the cylindrical coordinate system as follows

\[
\begin{align*}
\mathbf{H}(\rho, \theta, z) &= \mathbf{H}_b(\rho, \theta, z) \\
&+ \int_{\Omega} \frac{[(\sigma + i\omega \varepsilon) - (\sigma_b + i\omega \varepsilon_b)]}{(\sigma + i\omega \varepsilon)} \mathbf{E}^M_b(\rho', \theta', z', \rho, \theta, z) \left( \nabla \times \mathbf{H} \right)_{\rho'} d\rho' d\theta' dz' \\
&+ \int_{\partial \Omega^M} \mathbf{E}^M_b \times \mathbf{H} \cdot d\mathbf{S} - \int_{\partial \Omega^M} \frac{1}{(\sigma + i\omega \varepsilon)} \mathbf{H}^M_b \times (\nabla \times \mathbf{H}) \cdot d\mathbf{S},
\end{align*}
\]

where

\[
\nabla \times \mathbf{H} = \nabla \times \mathbf{H}(\rho', \theta', z'),
\]

\[
\nabla \times \mathbf{H}(\rho, \theta, z) = \frac{1}{\rho} \left| \begin{array}{ccc}
\rho & \rho \theta & \rho z \\
\rho \theta & \rho & \theta \\
\rho z & \theta & \rho
\end{array} \right|,
\]

\(E\) is the electric field, \(H\) is the magnetic field, \(E^M_b\) and \(H^M_b\) are \(3 \times 3\) Green’s tensor function excited by the magnetic dipole source. \(E^M_b(r', r)\) has integrative singular at \(r = r'\), and \(r = (\rho, \theta, z)\) locates in the outside boundary of the strip or in the center with \(\rho = 0\). The \(r'\) locates in \(\partial \Omega^M\), the internal boundary of the strip, therefore, the 3D strip magnetic field differential integral equation has no coordinate singular at the pole \(\rho' = 0\). It only has weak integrative singular kernel.

2.2. The 3D Magnetic Field Galerkin Equation

The 3D magnetic field Galerkin equation in the rectangular coordinate system is as follows

\[
\int_{\Omega^M} \left[ \left( \frac{1}{\sigma + i\omega \varepsilon} \nabla \times \mathbf{H}(r) \right) \left( \nabla \times \phi \mathbf{I}(r) \right) + i\omega \mu \mathbf{H}(r) \phi(r) \right] dr = \int_{\Omega^M} \mathbf{S}(r) \phi(r) dr.
\]
By substituting the field and coordinate cylindrical transformation formulas into the above Galerkin equation (4), we have the 3D magnetic field Galerkin equation in the cylindrical coordinate [7, 8],

\[
\int_{\Omega} \frac{1}{\sigma + i\omega \varepsilon} \left( \left( \frac{\partial H_\rho}{\partial z} - \frac{\partial H_\theta}{\partial \theta} \right) \frac{\partial \phi}{\partial z} - \left( \frac{\partial}{\partial \rho} \rho H_\theta - \frac{\partial H_\rho}{\partial \theta} \right) \frac{1}{\rho^2} \frac{\partial \phi}{\partial \theta} \right) \rho d\rho d\theta dz \\
+ i\omega \int_{\Omega} \mu H_\rho \phi \rho d\rho d\theta dz = - i\omega \int_{\Omega} \mu M_\phi \rho d\rho d\theta dz,
\]

\[
\int_{\Omega} \frac{1}{\sigma + i\omega \varepsilon} \left( - \left( \frac{1}{\rho} \frac{\partial H_\rho}{\partial \theta} - \frac{\partial H_\theta}{\partial z} \right) \frac{\partial \phi}{\partial z} + \left( \frac{\partial}{\partial \rho} \rho H_\theta - \frac{\partial H_\rho}{\partial \theta} \right) \frac{1}{\rho^2} \frac{\partial \phi}{\partial \theta} \right) \rho d\rho d\theta dz \\
+ i\omega \int_{\Omega} \mu H_\theta \phi \rho d\rho d\theta dz = - i\omega \int_{\Omega} \mu M_\phi \rho d\rho d\theta dz,
\]

\[
\int_{\Omega} \frac{1}{\sigma + i\omega \varepsilon} \left( \left( \frac{1}{\rho} \frac{\partial H_\rho}{\partial \theta} - \frac{\partial H_\theta}{\partial z} \right) \frac{1}{\rho} \frac{\partial \phi}{\partial \theta} - \frac{\partial \phi}{\partial \rho} \left( \frac{\partial H_\rho}{\partial z} - \frac{\partial H_\theta}{\partial \rho} \right) \right) \rho d\rho d\theta dz \\
+ i\omega \int_{\Omega} \mu H_\phi \phi \rho d\rho d\theta dz = - i\omega \int_{\Omega} \mu M_\phi \rho d\rho d\theta dz,
\]  

(5)

where \( \phi \) is a trilinear basic test function.

2.3. 3D AGILD EM Field Modeling

The 3D cylindrical domain is divided into a set of cylindrical cube elements. Each element has 8 nodes \((\rho_j, \theta_j, z_j), j = 1, 2, \ldots, 8\). The magnetic field differential integral equation (1) on the strip boundary zone and the center cylindrical sub domains and the Galerkin equations in the internal sub domains are coupled to construct the 3D AGILD EM field modeling [7, 8].

3. 3D AGILD EM INVERSION

3.1. The 3D Strip EM Parameter Variance Differential Integral Equation

The 3D magnetic field strip differential integral equation (1) in the cylindrical coordinate is the second type of integral equation for the magnetic field. From equation (1), we can derive the 3D strip EM parameter variance differential integral equation

\[
\delta \mathbf{H}(\rho, \theta, z) = \int_{\partial \Omega} \frac{\delta(\sigma + i\omega \varepsilon)}{\sigma + i\omega \varepsilon} \mathbf{H}_b^M \times (\nabla \times \mathbf{H}) \cdot d\mathbf{S} \\
- \int_{\Omega} \frac{\delta(\sigma + i\omega \varepsilon)}{(\sigma + i\omega \varepsilon)^2} \mathbf{E}_b^M (\rho', \theta', z', \rho, \theta, z) (\nabla \times \mathbf{H}) \rho' d\rho' d\theta' dz',
\]

(6)

where \( \delta \mathbf{H} \) is the data misfit. \( \delta \sigma \) is the conductivity variance, and \( \delta \varepsilon \) is the dielectric variance. Equation (6) is the first type of integral equation for the conductivity and dielectric variances and is of the ill posed condition.

3.2. The 3D EM Parameter Variance Galerkin Equation

From the 3D magnetic field Galerkin equation (4), the 3D EM parameter variance Galerkin equation can be derived as follows,

\[
\int_{\Omega} \left( \frac{\delta(\sigma + i\omega \varepsilon)}{(\sigma + i\omega \varepsilon)^2} \nabla \times \mathbf{H}(r) \right) (\nabla \times \phi \mathbf{I}(r)) dr = \\
\int_{\Omega} \left( \left( \frac{1}{(\sigma + i\omega \varepsilon)} \nabla \times \delta \mathbf{H}(r) \right) (\nabla \times \phi \mathbf{I}(r)) + i\omega \mu \delta \mathbf{H}(r) \phi(r) \right) dr,
\]

(7)

where \( \delta \sigma \) is the conductivity variance, \( \delta \varepsilon \) is the dielectric variance, \( \delta \mathbf{H} \) is the magnetic field variance, and \( r = (\rho, \theta, z) \). The weak ill posed variance Galerkin equation (7) is for the conductivity and dielectric variances.
3.3. 3D AGILD EM Inversion

The 3D strip EM parameter variance differential integral equation (6) on the boundary strip and central cylindrical hole and the 3D EM parameter variance Galerkin equation on the internal subdomains are coupled to construct the 3D AGILD EM inversion [7].

4. 3D-2D AGILD MULTIPLE CROSS HOLES INVERSION

4.1. The Multiple Cross Holes’ Data Configuration

In the geophysical exploration, oil exploration, and laser nanometer material detection, we describe the multiple cross holes’ data configuration in the Figure 1 and Figure 2. The sources are excited in the center hole and the receivers are placed in the surrounded holes to receive the EM data. The receiver holes can be numbered. The sources and receivers data configuration is shown in Figure 2. The data configuration generates multiple cross hole sections, as shown in Figure 1.

![Figure 1: The multiple cross holes cylinder.](image1)

![Figure 2: Multiple cross holes data configuration.](image2)

4.2. 3D-2D AGILD Multiple Cross Holes Inversion

To reduce the computational time cost, we propose the 3D-2D AGILD multiple cross holes inversion in this section. Because the transmitting sources are excited in the center cylindrical hole and data are received in the surrounded holes, we restrictively map the 3D AGILD EM inversion onto the 2D AGILD EM inversion on the three adjoining section sub domains with \( \theta_{j-1}, \theta_j, \theta_{j+1} \). Let \( AGI(\delta\sigma, \delta\varepsilon; H, \delta H, \theta) \) be the AGILD inverse operation for the increment of conductivity \( \delta\sigma \) and dielectric \( \delta\varepsilon \), where the magnetic field \( H \) and its variance \( \delta H \) are given by previous iteration, and \( \theta \) denotes the cross hole section with space variable \( \rho \) and \( z \). By induction, suppose the \( \delta\sigma_n \) and \( \delta\varepsilon_n \) in the \( \theta_{j-1} \) cross hole section are obtained in this iteration, the \( \delta\sigma_{n-1} \) and \( \delta\varepsilon_{n-1} \) in the \( \theta_{j+1} \) cross hole section are obtained by the previous \((n-1)\)th iteration. To find the \( \delta\sigma_n \) and \( \delta\varepsilon_n \) in the \( \theta_j \) cross hole section in Figure 1, we solve the 3D-2D AGILD inversion

\[
AGI(\delta\sigma_n, \sigma_n; H_{n-1}, \delta H_{n-1}, \theta_j) + AGI(\delta\sigma_n, \sigma_n; H_{n-1}, \delta H_{n-1}, \theta_j) + AGI(\delta\sigma_{n-1}, \delta\varepsilon_{n-1}; H_{n-1}, \delta H_{n-1}, \theta_{j-1}) = 0,
\]

5. THE 3D-2D AGILD MULTIPLE CROSS HOLES’ IMAGE

The cylindrical domain is divided into \( N \) cross holes sections. The 3D-2D AGILD EM inversion is used to recover the electric conductivity, dielectric, and magnetic permeability. In the strip boundary zone and center hole sub domain (Figure 1), we use collection Finite Element Method (CFEM) to discrete the magnetic field and parameter variance differential integral equation. The Galerkin FEM (GFEM) is used to discrete the Galerkin magnetic field and parameter variance equation in the internal sub domains. The strip boundary zone, center hole zone, and internal
discrete FEM equation are coupled to construct the 3D-2D AGILD discrete inversion equation,

\[
AGI \left( \delta \sigma_n^h, \delta \varepsilon_n^h; H_{n-1}^h, \delta H_{n-1}^h, \theta_{j-1} \right) + AGI \left( \delta \sigma_n^h, \delta \varepsilon_n^h; H_{n-1}^h, \delta H_{n-1}^h, \theta_j \right) + AGI \left( \delta \sigma_{n-1}^h, \delta \varepsilon_{n-1}^h; H_{n-1}^h, \delta H_{n-1}^h, \theta_{j-1} \right) = 0,
\]

where \( \delta \sigma_n^h \) is the piecewise FEM discretization of the \( \delta \sigma_n \) in the \( n \)th iteration, the \( \delta \varepsilon_n^h, H_{n-1}^h, \) and \( \delta H_{n-1}^h \) are the discretization of the \( \delta \varepsilon_n, H_{n-1}, \) and \( \delta H_{n-1}, \) respectively.

There are only unknown conductivity variance \( \delta \sigma_n^h \) and \( \delta \varepsilon_n^h \) in the cross hole section \( \theta_j \) to be solved. There are \( 12 \times 16 \), i.e., 172 elements in each cross hole section. The strip boundary zone full matrix and internal sparse matrix are fastly solved by the AGILD algorithm [7, 8]. After a few iteration steps of the 3D-2D AGILD inversion, we obtained reasonably high resolution cross hole images. The resistivity image in the cross sections 1, 3, 5, and 7 is plotted in Figures 4, 6, 8, and 10, respectively. The synthetic model is plotted in the left graph, the 3D-2D AGILD cross holes’ image is plotted in the right graph. The magnetic field data in the cross sections 1, 3, 5, and 7 are plotted in the Figures 3, 5, 7, and 9, respectively, the amplitude of the field is plotted in the left graph, and the phase of the field is plotted in the right graph. The two frequencies are 9600 Hz and 12000 Hz. The regularizing parameter in this paper is 1.9835. For some field data, the regularizing parameter will be larger than 2.
6. CONCLUSION

Many synthetic model datas and some field datas are interpreted by the 3D-2D AGILD multiple holes inversion. These images show that the 3D-3D AGILD multiple holes inversion method is fast, stable and has reasonably high resolution. The resistivity images in Figures 4, 6, 8, and 10 have obvious space variation in the $\theta$ direction, which shows the 3D-2D AGILD multiple inversion method has high resolution in the 3D space. The 3D-2D parallel AGILD inversion algorithm validation and the multiple cross holes imaging are performed by using 3D-2D AGILD software 32DAGILDINV which is made by Lee Xie in GLGEO. The cross checking validation between the 3D-2D AGILD inversion and GL modeling and inversion [11, 12] shows that 3D AGILD, 3D-2D AGILD, and GL EM modeling and inversion softwares are useful for oil exploration, earthquake exploration, geophysical engineering, environment characteristic monitoring, nondestructive testing, medical imaging, and material and nanometer materials sciences and engineering.

REFERENCES