A Fast Volume-surface Integral Equation Solver for Scattering Properties of NIMs

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Abstract—This paper presents a fast hybrid volume-surface integral equation approach for the computation of electromagnetic scattering from composed negative index media (NIM) such as split-ring-resonators (SRR) with wires. The volume electric field integral equation (EFIE) is applied to the dielectric region of this NIM, and the surface electric field integral equation is applied on the conducting surface. The method of moments (MoM) is used to discretize the integral equation into a matrix solution and adaptive integral method (AIM) is employed to reduce the memory requirement and CPU time for the matrix solution. The present approach is sufficiently versatile in handling scattering problems of composed NIMs, due to the combination of surface and volume electric field integral equations. Numerical results of calculating radar cross section (RCS) of such a NIM slab are finally presented to demonstrate the accuracy and efficiency of this technique.

DOI: 10.2529/PIERS061003050010

NIM, which is also known as left-handed material (LHM) [1], presents dielectric constants (permeability and permittivity) simultaneously negative. Typical NIMs, such as SRR [2], are composed of dielectric body and conducting patches. Based on this feature, to investigate the scattering problem of such NIMs, we can employ volume EFIE to the dielectric region and surface EFIE on the conducting surface [3]. Previous researchers often use MoM [4] to discretize the integral equations. And AIM [5, 6, 3] has already been proved to be an efficient solver in reducing the memory requirement for storage and to speed up the matrix-vector multiplication in the iterative solver.

For an SRR structure, in the dielectric region $V$, by taking the scattered field from both the surface current and volume current into consideration, the total electric field becomes:

$$E(r) = E^{\text{inc}}(r) + E^{\text{Sca}}(r)$$  (1)

Since the tangential components of total electric field vanish on conducting surface, we get:

$$\hat{n} \times E^{\text{inc}}(r) = -\hat{n} \times E^{\text{Sca}}(r)$$  (2)

Equations (1) and (2) are known as the EFIE as the formulations involve only electric field. EFIE is suitable for open conducting surface. Inside the dielectric region $V$ and on the surface of conducting body $S$, the incident wave induces volume current $J_V$ and surface current $J_S$. The induced volume and surface currents will generate scattered EM field as following:

$$E^{\text{Sca}}_{\Omega}(r) = -jk_0\eta_0 A_{\Omega}(r) - \nabla \Phi_{\Omega}(r), \quad \Omega = S \text{ or } V$$  (3)

where the magnetic vector potential is defined as:

$$A_{\Omega}(r) = \int_{\Omega} J_{\Omega}(r, r') g(r, r')dr', \quad \Omega = S \text{ or } V$$  (4)

and the electric scalar potential is defined as:

$$\Phi_{\Omega}(r) = -\frac{\eta_0}{jk_0} \int_{\Omega} \nabla' \cdot J_{\Omega}(r, r') g(r, r')dr', \quad \Omega = S \text{ or } V$$  (5)

where $g(r, r') = \frac{e^{-jk_0|r-r'|}}{4\pi |r-r'|}$, $\eta_0 = \sqrt{\mu_0 \varepsilon_0}$, and $k_0$ denotes the wavenumber of background medium (free space).

The volume of dielectric material and surface of conducting body are meshed into tetrahedral elements and triangular patches, respectively. These elements are used because of their flexibility.
to model arbitrarily shaped 3-D object. The volume and surface current are expanded using
different vector basis functions. For surface elements, it is convenient to use the planar triangular
basis functions or Rao-Wilton-Glisson (RWG) basis functions [7] to expand the equivalent surface
electric current. For volume elements, similarly, we can apply Schaubert-Wilton-Glisson (SWG)
basis functions [8] to expand the equivalent volume electric current.

\[ J_S = \sum_{n=1}^{N_S} I_n^S f_n^S \]

\[ J_V = j\omega \sum_{n=1}^{N_V} \frac{\tilde{\varepsilon}(r) - \varepsilon_0}{\tilde{\varepsilon}(r)} I_n^V f_n^V = j\omega \sum_{n=1}^{N_V} \kappa(r) I_n^V f_n^V \]

where \( \kappa(r) = \frac{\tilde{\varepsilon}(r) - \varepsilon_0}{\tilde{\varepsilon}(r)} \) is the contrast ratio and \( \tilde{\varepsilon}(r) \) is the permeability of a tetrahedron element.

After substituting above equations into EFIE, we applying the Galerkin’s testing procedure.
Then the integral equations are converted into a linear equation system written as:

\[
\begin{bmatrix}
\bar{Z}^{VV} & \bar{Z}^{VS} \\
\bar{Z}^{SV} & \bar{Z}^{SS}
\end{bmatrix}
\begin{bmatrix}
I^V \\
I^S
\end{bmatrix}
= \begin{bmatrix}
E^V \\
E^S
\end{bmatrix}
\]

where the vectors \( I^V \) and \( I^S \) represent the coefficients of volume current and surface current re-
spectively. The excitation vector can be computed using

\[ E_m^V = \int_{V_m} f_m^V \cdot E^{inc}(r')dr' \]

\[ E_m^S = \int_{S_m} f_m^S \cdot E^{inc}(r')dr' \]

Then, we apply AIM and decompose \( \bar{Z} \):

\[ \bar{Z} I = \bar{Z}^{near} I + \bar{Z}^{far} I \]

where \( \bar{Z}^{near} \) is a sparse matrix that contains only the nearby elements within a threshold distance and can be compute with iteration method. \( \bar{Z}^{far} \) represents the far-zone interaction of the elements. We apply FFT to \( \bar{Z}^{far} I \) in order to make a good approximation in the far-zone:

\[ \bar{Z}^{far} I = \tilde{\Lambda} \tilde{\Im}^{-1} \{ \Im\{\tilde{g}\} \cdot \Im\{\tilde{A}^T I\} \} \]

where \( \tilde{\Im}\{\bullet\} \) and \( \tilde{\Im}^{-1}\{\bullet\} \) stand for FFT and inverse FFT respectively. The Matrix \( \tilde{g} \) is Toeplitz. \( \tilde{\Lambda} \) represents the basis transformation matrix of the elements.

A direct solver of MoM requires \( O(N^3) \) operations to solve the equation while an iterative solver requires \( O(N^2) \) operations in each iteration. Both solvers require \( O(N^2) \) memory to store the matrix elements. However, the computational complexity for AIM is \( O(N1.5\log N) \) and \( O(N\log N) \) for surface and volume scatterers, respectively.

Figure 1: Geometry of a single inclusion-SRR and Wire (d1=2.63 mm, d2=1.53 mm).
Numerical Result
First, we work with one row of SRR structure (3 inclusions placed side by side along \( y \)). The geometry and dimensions of this kind of inclusions are shown in Figure 1. The width of all metal strips is 0.25 mm, the thickness of dielectric is 0.254 mm and the length of each square is 3.3 mm. The relative permittivity of dielectric is set to be \( \varepsilon = 1 \).

![Figure 2: SCS versus frequency for a row of inclusions (3 SRRs) at \( \theta = \phi = 90^\circ \).](image)

We examine the scattering cross section (SCS) \([9]\) of this row of 3 elements and plot it verses frequency in Figure 2. It can be seen that the resonant frequency is approximately 15.80 GHz.

![Figure 3: Structure of the slab composed by SRRs and wires with two different incidents.](image)

Next, at this resonant frequency, we analyze the propagation characteristics of electromagnetic wave in a NIM sample placed in the free space. It’s a NIM slab composed of many rows of inclusions shown in Figure 1. The 3-D view of this slab is depicted in Figure 3. The space distances of the inclusions denoted, respectively, by \( d_x \), \( d_y \) and \( d_z \) in the \( x \)-, \( y \)- and \( z \)-directions, are all 3.3 mm. There are totally 54 SRR elements which are arranged into 18 rows along \( x \)- and 3 columns along \( y \)-. In order to show that our AIM algorithm is suitable for this structure, we illuminate the slab with two different plane waves coming from \(-y\) direction as shown in Figure 3, and then we check the RCS of this SRR slab in each case. Actually, SRR requires the electric field \( E^{inc} \) in parallel with the plane of the ring \((yOz\) plane\) to get maximum magnetic resonant. In other words, the rings are not supposed to be on the \( H - k \) plane \([10]\). So if \( E^{inc} \) is perpendicular with the plane of rings (case (2) in Figure 3), the RCS must be very small and the whole structure becomes no more
than an ordinary scatterer which does not acquire negative refractive index. Figure 4 and Figure 5 show values of $E_z$ and RCS when the illuminators are of case (1) and case (2), respectively.

Figure 4: $E_z(r,t)$ and RCS of the slab of case (1) in Figure 3 at 15.8 GHz.

Figure 5: $E_z(r,t)$ and RCS of the slab of case (2) in Figure 3 at 15.8 GHz.

Conclusions

The structures of SRR with wires behave as a magnetic conductor in the vicinity of resonant frequency. By applying the fast solver AIM, the CPU time per iteration as well as memory requirement is greatly reduced. SRR structures are often electrically small and have a large number of unknowns, so AIM is especially suitable to analyze such kind of NIMs. We first use a few inclusions to find the resonant frequency, then at the vicinity of this frequency, an SRR slab is tested when the illuminator are two different plane waves. When the $E$ is in parallel with the ring, most power can be scattered which satisfies the property of NIM. However, when $E$ is perpendicular with the ring, there is neither magnetic resonance nor negative refractive index.

REFERENCES


