Debye Series Analysis of Forward Scattering by a Multi-layered Sphere

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Abstract—A formula of Debye series for forward scattering by a multi-layered sphere is presented. The Debye series expansion allows for the decomposition of the global scattering process in a series of local interactions, and clarifies the physical origins of many effects that occur in electromagnetic scattering. The forward-scattering pattern contains many information about the properties, and the method is widely used for determining such properties on the basis of forward-scattering pattern. In the paper, the Debye series is employed to the study of forward scattering by a multi-layered sphere, which is of great importance to the study of characteristics of particles.

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For many practical applications such as combustion, environmental control, fluid mechanics, and chemical reaction, we need to measure the properties of particles such as size and refractive index-temperature ratio quickly and precisely. The forward-scattering pattern contains many information about the properties, and it is widely used for determining such properties on the basis of forward-scattering pattern.

The scattering of a plane electromagnetic wave by a single multilayered spherical particle has been extensively discussed within a theoretical framework similar to the Lorenz-Mie Theory (LMT) in many areas of theoretical and applied research, such as combustion, chemical engineering, remote sensing, communication, biology, and medicine [1]. The Mie theory is a rigorous solution of the Maxwell equations and contains all effects that contribute to the scattering [2, 3]. But it gives few clues to the various physical processes that are responsible for the scattering [2–4]. Full geometrical optics theory can be used with reasonable accuracy for forward direction light scattering computations related to particle sizing and characterization [5], and has been extended to forward scattering by coated particles [6]. But as soon as the scatters are complicated, for instance multi-layered sphere, the construction of the geometrical optics approximation formula is very difficult.

The Debye series writes each term of the Mie series as another infinite series, and clarifies the physical origins of many effect that occur in electromagnetic scattering [2, 3], which is of great importance in the study of characteristics of electromagnetic scattering. The Debye series was originally formulated for scattering of a normally incident plane wave by a cylinder [7] and has been subsequently extended to scattering by a sphere [8, 9], the internal fields [10], scattering by a coated sphere [11], scattering of a plane wave diagonally incident on a cylinder [12], and scattering by a multilayered sphere [2] and a spherical Bragg grating [13].

We consider an $l$-layered dielectric sphere whose refractive index of any layer $j$ is $m_j$ (region $j$) and whose radius $a$ is embedded in a dielectric medium of refractive index $m_{l+1}$ (region $l+1$), as shown in Fig. 1. When the sphere is illuminated by a monochromatic plane wave of wavelength $\lambda$, the classic Mie coefficients $a^l_n$ and $b^l_n$ can be expanded in the Debye series [2]

$$ a^l_n = \frac{1}{2} (1 - Q^l_n) \quad (1) $$

where $Q^l_n$ for region $j$ can be written as:

$$ Q^l_n = R^{j+1,j,j+1}_n + T^{j+1,j}_n Q^{-1}_{n+1} T_{n+1,j} + \sum_{p=1}^{\infty} (R^l_n Q^{-1}_n T^{j+1,j}_n Q^{-1}_{n+1})^{p-1}, \quad (2) $$

The coefficients $R^{j+1,j,j+1}_n, T^{j+1,j}_n, T^{j+1,j}_n, R^l_n$ are depicted in Fig. 1, and have the same definitions as in Ref. [2]. $P$ is the mode of refraction, and depicted by Fig. 1 in Ref. [2]. When summed
over \( n \), the first term \( 1/2 \) in Eq. (1) corresponds to diffraction, and the second term \(-1/2R_{n+1}^{j+1,j,j+1}T_{n+1}^{j+1,j,j+1}\) to the reflection waves. The third term is a series that describes the contributions of all modes of refraction. Each term in the series represents the contribution of that mode that has undergone \( p - 1 \) internal reflections and then emerged from the sphere. The main advantage of Debye series is to isolate the single mode.

As shown in Ref. [14], upward of 99.5% of the total forward-scattered light for both polarizations emerges from the first interface after simple reflection and from the second interface after twofold refraction. For the forward scattering, the each partial-wave scattering amplitude can be written as a sum of diffraction of the partial wave, its external reflection from the sphere surface, and direct transmission through the sphere. Mathematically speaking, with the help of Debye series, the scattering coefficients \( a_n^l \) and \( b_n^l \) can be written as

\[
\frac{a_n^l}{b_n^l} = \frac{1}{2} \left( 1 - R_{n+1}^{j+1,j,j+1}T_{n+1}^{j+1,j,j+1} - Q_{n+1}^{j+1,j,j+1}T_{n+1}^{j+1,j,j+1} \right) \quad (3)
\]

When summed over \( n \), the first term \( 1/2 \) in Eq. (3) corresponds to diffraction, the second term \(-1/2R_{n+1}^{j+1,j,j+1}T_{n+1}^{j+1,j,j+1}\) to the reflection ray, and the third term \(-1/2T_{n+1}^{j+1,j,j+1}Q_{n+1}^{j+1,j,j+1}T_{n+1}^{j+1,j,j+1}\) to the ray penetrating the sphere and emerging from the sphere without internal reflection.

For clarity in the figure, we consider a sphere of relatively small radius \( a = 10 \mu m \) stratified in 100 layers illuminated by a plane wave of wavelength \( \lambda = 632.8 \) nm. The refractive index profile of the sphere is depicted in Fig. 2.

It is depicted in Fig. 3 the scattered intensities of diffraction, diffraction including reflection, diffraction including reflection and direct transmission, as well as the Mie scattered intensity. The refractive index of the sphere is depicted in Fig. 2.

From Fig. 3, we can find that diffraction of incident wave around the sphere is the main fraction of forward scattered intensity, and in the very small angle region, the intensity of pure diffraction or diffraction including external reflection agrees with Mie scattered intensity well. But with the angle increasing, the difference between them increases. Pure diffraction intensity or the interference intensity of diffraction including external reflection is not sufficient. If the wave of direct transmission through the sphere is considered additionally, the intensity agrees with that by Mie theory well in a wider range of angles.

\( Q_{n+1}^{j+1,j,j+1} \) in Eq. (3) gives the influence of the whole core (from center to region \( j - 1 \)) on the composite sphere scattering. If Eq. (3) is written as

\[
\frac{a_n^l}{b_n^l} = \frac{1}{2} \left( 1 - R_{n+1}^{j+1,j,j+1}T_{n+1}^{j+1,j,j+1} - Q_{n+1}^{j+1,j,j+1}T_{n+1}^{j+1,j,j+1} \right) \quad (4)
\]

then such influence of the core is excluded. It is depicted in Fig. 4 the comparison of scattered intensity simulated by Eq. (3) with that by Eq. (4). From the figure, we can find that if the
influence of the core is included, a shift of angle position appears between two curves. This is because if the influence of the core is excluded, the sphere can be considered as a homogeneous sphere whose refractive index equals to the refractive index of layer \( l \). Once the influence is included, the refractive index profile of sphere changes, and a shift of angle position will appear.

The forward-scattering pattern contains much information about the properties, and the separation of such information is of great practice. Debye series has the advantage of the decomposition of global scattering process in a series of local interaction, and can be employed to the simulation of scattered intensity of single mode or interference intensity of many modes. In this paper, a formula of Debye series expansion of forward scattering by multi-layered sphere is presented, and is employed to the study of forward scattering by multi-layered sphere. The construction of forward scattered intensity is studied on the use of Debye series, and the influence of the core on composite sphere scattering is discussed as well.

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REFERENCES