Application of the Array Scanning Method to 1D-periodic Microstrip Lines

R. Rodríguez-Berral¹, F. Mesa¹, P. Baccarelli², and P. Burghignoli²

¹Department of Applied Physics 1, University of Seville, Av. Reina Mercedes s/n, 41012 Seville, Spain
²Department of Electronic Engineering, “La Sapienza” University of Rome
Via Eudossiana 18, 00184 Rome, Italy

Abstract—The Array Scanning Method (ASM) is employed to analyze the excitation of 1D-periodic microstrip lines by a nonperiodic source. It should be noted that the nonperiodic nature of the source precludes the direct application of Floquet’s theorem. However, a combination of the ASM with the Method of Moments allows for the computation of the current density launched on the line. Some results are presented showing the current excited by a delta-gap voltage source at frequencies corresponding to passband as well as to stopband regimes of the periodic microstrip line.

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Planar printed structures with periodicity along one or two dimensions (1D- or 2D-periodic structures, respectively) are of great interest in their application to the design of passive microwave circuit components such as filters, EBG’s, and leaky-wave antennas with scanning capabilities. Most of the previous studies devoted to these structures have been restricted to the computation of the propagation characteristics of their modal solutions (Bloch waves or Floquet modes) to find, for example, the different frequency passbands and stopbands and the radiative regimes.

This work proposes an efficient approach to deal with the excitation of 1D-periodic microstrip lines by a single nonperiodic source. The presence of a nonperiodic source breaks the periodicity of the problem, thus causing the usual techniques based on the direct application of Floquet’s theorem not to be applicable. This fact notably increases the degree of difficulty of the problem under consideration with respect to the modal analysis of the periodic structure that is being excited. As it will be explained, the formulation presented here to overcome this difficulty relies on the fact that the current excited by the nonperiodic source can be expressed as a continuous superposition of Floquet-periodic functions that are solutions to auxiliary Floquet-periodic problems. For simplicity, a simple canonic structure will be considered, which consists of a uniform microstrip line periodically loaded with gaps (or, equivalently, an infinite 1D array of rectangular metallic patches) and excited by a delta-gap voltage source. In order to solve the above-mentioned auxiliary problems, the method of moments is employed.

Our goal is the computation of the current excited on a periodic microstrip line when it is excited by a delta-gap voltage source. Specifically, the structure under study (see Fig. 1) is an

![Figure 1: Top and side views of the structure under analysis.](image-url)
array of identical and perfectly conducting rectangular patches that extends indefinitely along the \( \hat{z} \) direction; namely, a 1D-periodic structure whose unit cell has length \( p \). The patches are printed on a lossless isotropic dielectric substrate of thickness \( h \). This structure is excited by a delta-gap voltage source of finite length \( \delta \) centered on one of the patches, which in the following will be referred to as the central patch. This excitation can be modelled as an impressed electric field in the gap region, namely

\[
E(x, z) = E_g(z)\hat{z}, \quad E_g(z) = -V_g \text{rect} \left( \frac{z}{\delta} \right).
\]  

(1)

It is important to note that the presence of this impressed field causes the problem to be non-periodic, thus preventing the direct application of the usual techniques based on Floquet’s theorem. Nevertheless, the gap field can be expressed in terms of the following integral:

\[
E_g(z) = \frac{p}{2\pi} \int_{-\pi/p}^{\pi/p} E^\infty_g(z; k_z) dk_z,
\]

where

\[
E^\infty_g(z; k_z) = \sum_{n=-\infty}^{\infty} E_n(z - np)e^{-j k_z np}.
\]

(2)

Note that \( E^\infty_g(z; k_z) \) is a Floquet-periodic function of \( z \) with period \( p \), i.e., a function such that \( E^\infty_g(z + p; k_z) = e^{-jk_zp}E^\infty_g(z; k_z) \). Thus, the problem defined by the excitation of the periodic structure depicted in Fig. 1 by an impressed field of the form \( E^\infty_g(z; k_z) \) turns into a Floquet-periodic problem that can be readily solved by applying Floquet’s theorem. Once this auxiliary Floquet-periodic problem is solved, the solution to the original nonperiodic problem can be found by superposition making use of (2). This technique, in which the original problem is decomposed into a continuous superposition of periodic problems, is known in the literature as Array Scanning Method [1, 2].

Let us consider a field of the form (3) impressed on the periodic array of patches depicted in Fig. 1. As the patches are perfect conductors, the tangential electric field on their surface must equal the impressed field. Hence, the following integral equation must be satisfied on the surface of all the patches:

\[
\hat{y} \times \mathbf{E}[\mathbf{J}^\infty] = E_g^\infty \hat{x},
\]

(4)

where \( \mathbf{J}^\infty \) is the surface current density on the patches. In order to apply the method of moments to equation (4), the current density is next expanded into a linear combination of basis functions. For simplicity, only the longitudinal component of the current density will be considered, as well as only one basis function for its dependence on the \( x \) coordinate. Although the integral equation (4) must be enforced on all the patches, it is sufficient to enforce it on one single patch due to the Floquet-periodic nature of the problem.

Next, the integral equation is solved on the central patch by applying the method of moments in a Galerkin formulation. Taking now into account that the original nonperiodic excitation was written as a combination of Floquet-periodic impressed fields by means of the linear transformation in (2), after applying the superposition principle, the current density excited by the delta-gap source on the periodic microstrip line can be finally computed as

\[
J(x, z) = \frac{p}{2\pi} \int_{-\pi/p}^{\pi/p} J^\infty(x, z; k_z) dk_z
\]

(5)

Next, some results obtained by using the proposed formulation are shown. Fig. 2 shows the amplitude of the current excited by the delta-gap source on the periodic microstrip line in Fig. 1 at 13 GHz (see the caption for the details of the structure). This value of frequency is located within a passband region since the periodic microstrip line has one real bound mode (BM) above cutoff, whose fundamental spatial harmonic wavenumber is \( k_z^{BM} = 1.63k_0 \). Far enough from the central patch, the current on the line is accounted for basically by the bound-mode current, since it will be the only non-attenuating component of the current.

Figure 3 shows the current excited on the same periodic microstrip line at 15 GHz. Unlike the passband case above, this value of the frequency is located within the first stopband of the periodic microstrip line. The wavenumbers of the spatial harmonics that constitute the BM are
Figure 2: Amplitude of the current excited by a delta-gap voltage source of length $\delta = 0.1\,\text{mm}$ on a periodic microstrip line as in Fig. 1 with $p = 4\,\text{mm}$, $L = 3.8\,\text{mm}$, $w = 0.6\,\text{mm}$, $h = 0.767\,\text{mm}$, and $\varepsilon = 10.2\varepsilon_0$. The frequency is 13 GHz.

now complex (all of them having the same imaginary part) and lie on the boundary of the Brillouin zones. Specifically, the wavenumber of the fundamental harmonic is $k_{BM} = \pi/p - j0.87k_0$. As a consequence, the BM fields and current decay exponentially in $z$. It is interesting to note that in this case the BM fields do not carry power but they are a purely reactive standing wave.

Figure 3: Amplitude of the current excited on the same periodic microstrip line of Fig. 2. The frequency is now 15 GHz.

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