Adaptive Selection of Sampling Interval In Frequency Domain for Estimating the Poles of Scatterers

Shaogang Wang, Xinpu Guan, Dangwei Wang, Xingyi Ma, and Yi Su
School of Electronic Science and Technology
National University of Defense Technology, Changsha 410073, China

Abstract— In this paper, the matrix pencil (MP) method has been utilized for estimating the complex natural resonances from IFFT of calculated frequency responses. The novelty is that frequency sampling interval is investigated. The responses are usually computed by the accurate method of moments (MoM) in frequency domain, which is time-consuming. So, the frequency sampling interval’s effect on results should be analyzed, and a scheme for adaptive choice of frequency sampling interval is proposed in this paper. The scheme is validated by a special example, a perfect conducting sphere. Lastly, poles of a complex radar target, a missile model, are calculated by the proposed method and compared with other available data.

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1. INTRODUCTION

It is known that extracting complex natural resonance frequencies (poles) of the target from its transient response is desired in target identification. The target poles contain the information of decay factors and the resonant frequencies that are only determined by target’s structure and material. So, they should be estimated with high accuracy. Of many methods, the matrix pencil (MP) is an effective one for extracting complex natural resonance frequencies of scatterers from transient responses [1–3].

A method in time domain, finite difference time domain (FDTD) or MoM in time domain (TD-MoM), can be used to calculate transient responses, from late time part of which poles are estimated. But that poles are not accurate as those extracted from impulse responses, which can be obtained by the deconvolution technology in time domain. Noteworthily, noise will be inflicted in deconvolution.

The method of moments (MoM) in frequency domain is widely used, accurate and chosen to calculate the frequency response in this paper. The impulse response can be obtained by using Inverse Fast Fourier Transform (IFFT) technique from the frequency response.

As all known, frequency sampling interval is less and more accurate results are obtained. On the other way, MoM is time-consuming. The sampling interval must be investigated for results with enough accuracy in a short period of time. An algorithm of adaptive choice of frequency sampling interval is proposed in this paper.

The paper is organized as follows. In the next section, the mathematical formulation of MP is briefly presented. In Section 3, a scheme for adaptive choice of frequency sampling interval is proposed and validated. Finally, in Section 4, the proposed scheme is validated through a perfect conducting sphere and an application is presented. Numerical results of a complex radar target, a missile model, are computed for poles and compared with available data.

2. FORMULATIONS OF THE MP METHOD

If the frequency response calculated by MoM is \([Y(f_i)]\) \((i = 0, 1, \ldots, N' - 1)\). The sampling interval is \(f_s\). Then, The impulse response, \([y(kT_s)]\) \((k = 0, 1, \ldots)\), can be obtained through IFFT. \(T_s\) is the sampling period, and \(T_s = 1/(2N'f_s)\). By the singularity expansion method (SEM), the time sequence can be rewritten as

\[
y(kT_s) \approx \sum_{i=1}^{M} R_i z_i^k \quad \text{for} \quad k = 0, 1, \ldots, N - 1
\]

and

\[
z_i = e^{s_i T_s} = e^{(-\alpha_i + j\omega_i) T_s} \quad \text{for} \quad i = 1, 2, \ldots, M
\]
where, \( s_i \) and \( R_i \) are poles and residues. \( N \) is the length of impulse response. \( M \) is the number of poles.

Two matrices \([Y_1]\) and \([Y_2]\) are considered, which defined as

\[
[Y_1] = \begin{bmatrix} y(0) & y(1) & \cdots & y(L-1) \\ y(1) & y(2) & \cdots & y(L) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L-1) & y(N-L) & \cdots & y(N-2) \end{bmatrix}_{(N-L) \times L}
\]

\[
[Y_2] = \begin{bmatrix} y(1) & y(2) & \cdots & y(L) \\ y(2) & y(3) & \cdots & y(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L) & y(N-L+1) & \cdots & y(N-1) \end{bmatrix}_{(N-L) \times L}
\]

The pencil parameter, \( L \), is very useful in eliminating some effects of noise in the data.

The parameters \( z_i \) may be found as the generalized eigenvalues of the matrix pair \( \{[Y_2]; [Y_1]\} \). Equivalently, the problem of solving for \( z_i \) can be cast as an ordinary eigenvalue problem,

\[
\{[Y_1]^+[Y_2] - \lambda[I]\}
\]

where \([I]\) is identify matrix, and \([Y_1]^+\) is the Moore-Penrose pseudoinverse of \([Y_1]\).

The residues, \( R_i \), are solved for from the following least-squares problem:

\[
\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ Z_1 & Z_2 & \cdots & Z_M \\ \vdots & \vdots & \ddots & \vdots \\ Z_1^{N-1} & Z_2^{N-1} & \cdots & Z_M^{N-1} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix}
\]

3. ADAPTIVE CHOICE OF FREQUENCY SAMPLING INTERVAL

It is time-consuming to calculate frequency responses by MoM, so fewer frequency sampling points are expected. On the other way, it needs more frequency sampling points to obtain more accurate results. A scheme of adaptive choice of frequency sampling interval is proposed in this part.

Although a scatterer has many poles, only several of them (about five at most) can be numerically obtained and useful to characterize the target [4]. It is important to accurately estimate those dominant poles. So, an average relative error, \( err \), is given in advance and defined as

\[
err = \frac{\sum_{i=1}^{M'} |s_i - s'_i|}{\sum_{i=1}^{M'} |s_i|}
\]

where \( M' \) is the number of dominant poles. \( s'_i \) are estimated poles from frequency response at an interval of \( f_s \), and \( s_i \) are those at an interval of 0.5\( f_s \). If the value of (7) is less than the given reference error \( \varepsilon \), \( s_i \) are the expected results. Otherwise, the frequency interval is chosen as half of that last time, and poles are calculated again until the average relative error is less than \( \varepsilon \).

The procedure of the algorithm is following:

1. Give expected error \( \varepsilon \), frequency step \( f_s \), the number of frequency samples \( N' \).
2. Calculate frequency response, poles \( s'_i \) and estimate the number of dominant poles \( M' \).
3. Let \( f_s = 0.5f_s \), calculate frequency response, and poles \( s_i \).
4. Compute the relative error \( err \).
5. If \( err > \varepsilon \), let \( s'_i = s_i \), and go to (3).
6. If \( err < \varepsilon \), stop it.
4. NUMERICAL EXAMPLES

For the scheme validated, a conducting sphere, the radius of which is 0.06 m ($a = 0.06$), is specifically considered. The incident wave is plane wave of unit amplitude. A frequency range of 0 Hz (excluded) to 5.12 GHz is chosen as the excitation frequencies. At the observation 1000 m far from the center of the sphere, the backscattering response data, which can be calculated by MoM or Mie here, are used to extract poles.

In Table 1, poles are valued as the form of $sa/c$. $err_1$ are the average relative errors between numerical results and theoretical values. $err_2$ are the average relative errors between numerical results and those of last time. If given $\varepsilon = 0.01$, from Table 1 we can see that $f_s = 40$ MHz and only 128 frequency points are enough for accurate poles. The errors indicate the proposed scheme is correct.

<table>
<thead>
<tr>
<th>Theory [5]</th>
<th>$f_s = 160$ MHz $N' = 32$</th>
<th>$f_s = 80$ MHz $N' = 64$</th>
<th>$f_s = 40$ MHz $N' = 128$</th>
<th>$f_s = 20$ MHz $N' = 256$</th>
<th>$f_s = 10$ MHz $N' = 512$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.500+j0.866$</td>
<td>$-0.500+j0.854$</td>
<td>$-0.496+j0.858$</td>
<td>$-0.498+j0.862$</td>
<td>$-0.500+j0.864$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.702+j1.807$</td>
<td>$-0.668+j1.768$</td>
<td>$-0.700+j1.791$</td>
<td>$-0.702+j1.798$</td>
<td>$-0.700+j1.803$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.843+j2.758$</td>
<td>$-0.846+j2.636$</td>
<td>$-0.836+j2.736$</td>
<td>$-0.839+j2.747$</td>
<td>$-0.843+j2.752$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.954+j3.715$</td>
<td>$-1.096+j3.368$</td>
<td>$-0.948+j3.682$</td>
<td>$-0.951+j3.697$</td>
<td>$-0.953+j3.706$</td>
</tr>
<tr>
<td>5</td>
<td>$-1.048+j4.676$</td>
<td>$-1.003+j3.995$</td>
<td>$-1.047+j4.640$</td>
<td>$-1.049+j4.654$</td>
<td>$-1.048+j4.668$</td>
</tr>
<tr>
<td>6</td>
<td>$-1.129+j5.642$</td>
<td>$-0.900+j9.19$</td>
<td>$-1.122+j5.615$</td>
<td>$-1.109+j5.597$</td>
<td>$-1.114+j5.644$</td>
</tr>
</tbody>
</table>

$err_1 = 0.1$ $err_1 = 0.007$ $err_1 = 0.006$ $err_1 = 0.002$ $err_1 = 0.001$
$err_2 = 0.09$ $err_2 = 0.004$ $err_2 = 0.004$ $err_2 = 0.001$

As an application, poles of a common complex radar target are calculated in this part. A kind of missile model is considered as shown in Fig. 1(a) and Fig. 1(b). The unit of labels is meter. The main part of the model is a circle cylinder. The front part is half a spheroids, and the tail is half a sphere.

Table 2: Poles and relative errors under different frequency steps of the missile.

<table>
<thead>
<tr>
<th>MoM</th>
<th>$f_s = 2.4$ MHz $N' = 128$</th>
<th>$f_s = 1.2$ MHz $N' = 256$</th>
<th>$f_s = 0.6$ MHz $N' = 512$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.45+j2.37$</td>
<td>$-0.465+j2.318$</td>
<td>$-0.466+j2.355$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.34+j4.30$</td>
<td>$-0.355+j4.256$</td>
<td>$-0.356+j4.296$</td>
</tr>
</tbody>
</table>

$err_1 = 0.015$ $err_1 = 0.0057$ $err_1 = 0.006$
$err_2 = 0.01$ $err_2 = 0.004$ $err_2 = 0.005$

With the incident plane wave of unit amplitude z-polarized and x-traveling, the frequency responses at the observation 1000 m far from the center of the half sphere are calculated by a fast MoM method [6] in the range from 0 Hz (excluded) to 307.2 MHz. That obtained frequency responses are shown in Fig. 2. The cut impulse with 100 points ($N = 100$) is used. Applying the MP method with $L = 49$, we only obtain two stable dominant poles shown in Table 2. The poles are valued in the form as $sL'/c$, where $L' = 1.3$ m is the length of the model. The compared data are obtained using the approach as in [7]. The required relative error is also 0.01 ($\varepsilon = 0.01$). The meanings of $err_1$ and $err_2$ are the same as those in Table 1.

The errors in Table 2 show that the poles we calculated are correct at least frequency sampling points, and 512 points is enough.
Figure 1: (a) The planform of the missile model, (b) The side view of the missile model.

Figure 2: The frequency response of the model.

5. CONCLUSION

In application of the MP method for estimating the complex natural resonances of scatterers from frequency response data, an algorithm of adaptive choice of frequency sampling interval is proposed in this paper, because the responses are usually calculated by time-consuming MoM in frequency domain. Numerical examples show our method’s effect on the results with required precision from frequency points at least.

REFERENCES
