Longitudinal Power Distribution and Corresponding Temperature Distribution in a RF Waveguide CO₂ Laser

A. Rauf, Bingxin Zhang, and Jianguo Xin
Department of Physical Electronics, School of Information Science and Technology
Beijing Institute of Technology, Beijing 100081, China

Abstract—The output power of a gas laser can not be increased by increasing the input power due to the increase of the gas temperature. As the temperature of the laser gas mixture increases up to 600 K, the amplification of the amplifying medium decreases rapidly and so the output power decreases. In this paper, longitudinal power distribution and corresponding temperature distribution along the CO₂ waveguide laser are studied theoretically. Effect of voltage and current distributions along and transverse to the electrode direction respectively are accounted for the studies. It is seen that the temperature decreases exponentially along the length of the electrodes on both sides of the feed point.

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Several papers [1] have already described the huge potentiality of large area diffusion cooled RF capacitive discharges for the construction of medium — to high power CO₂ lasers for materials processing. Indeed, some industrial sources based on this technology are presently appearing [2]. However, there exist some open problems that limit the simple scaling of the above-mentioned technique to higher power segments. One of the major problem is certainly the attainment of a uniform gain medium excitation because of transmission line effects [3] naturally determined by electrode dimensions comparable with the exciting RF wavelength. Typical counter measures to this difficulty are the adoption of sectioned discharges or the use of reactive elements along the electrodes. The principles for the correct design of systems based on these schemes are well established in the case of narrow channel devices [4–6], in which the equivalent-line characteristic impedance is mainly determined by the electrode structure.

In [7] it is demonstrated that the same smoothing principals could be applied to wide channel high power discharges provided that the influence of the discharge loading be taken into account. Indeed, from the analysis carried out in [7], it is possible to conclude that the main role played by the discharge loading is that of determining a new equivalent transmission line whose characteristic impedance is closer to that produced by the capacitance of the ion sheaths surrounding the electrodes rather than by the channel capacitance. This effect determines the voltage and thus power distribution closer to a lossless-line rather than lossy-line models. Moreover [8, 9], the typical impedances of sheaths and neutral plasma for the mixtures and pressure of the interests for CO₂ laser construction produce distributions hardly distinguishable from uniform transmission line models [7, 10, 11].

In this paper the temperature distribution along the longitudinal direction of the electrodes is studied theoretically by considering the longitudinal power distribution by applying the transmission line theory. Voltage distribution is considered along the longitudinal direction while the current in transverse direction of the electrodes. The voltage and current distributions along the electrodes are given by solving the simple transmission line differential equations [2, 7, 10–12]:

\[
\frac{d^2 I(x)}{dx^2} - \gamma^2 I(x) = 0
\]

\[
\frac{d^2 V(x)}{dx^2} - \gamma^2 V(x) = 0
\]

where

\[
\gamma = \sqrt{ZY(x)} = \sqrt{(R + j\omega L)(G(x) + j\omega C)}
\]

And \( Z = R + j\omega L \) and \( Y(x) = G(x) + j\omega C \). Where \( L \) is the longitudinal inductance per unit length, \( C \) is the electrode’s structure capacitance per unit length and \( G(x) \) is the local plasma conductance.
We neglect the longitudinal resistance being mainly due to the metal electrodes and the values of \( L \) and \( C \) are adjusted by iteration. The general solutions of the Eqs. (1) and (2) are given by:

\[
I(x) = Ae^{\gamma x} + Be^{-\gamma x}
\]

\[
V(x) = Ce^{\gamma x} + De^{-\gamma x}
\]

where \( A, B, C, \) and \( D \) are the constants and can be determined by applying the suitable boundary conditions. Considering the feed point at the centre and applying the condition that at \( x = \pm d \): \( I = 0 \), where “\( d \)” is length of the electrode on one side of the central feed point, we have the relation for current distribution from Eq. (4) as:

\[
I(x) = (e^{\gamma x} - e^{-\gamma (x+2d)} + e^{-\gamma x} - e^{\gamma (x-2d)})
\]

Eq. (6) represents the current flowing along the longitudinal direction. In order to find the voltage distribution along the longitudinal direction, part of the current flowing transverse to the electrode is considered, i.e.,

\[
\frac{dI(x)}{dx} = -YV(x)
\]

From Eqs. (6) and (7) we have the relation for voltage distribution as:

\[
V(x) = \frac{r}{Y}(-e^{\gamma x} - e^{-\gamma (x+2d)} + e^{-\gamma x} + e^{\gamma (x-2d)})
\]

In real facts the discharge structure is still more complicated than the uniform transmission line model considered so far. Indeed the plasma conductivity is a function of the local voltage over the electrodes and thus cannot be considered uniform. The plasma conductance is given by [8, 9]:

\[
G_{pl} = \frac{1}{r_{pl} - \frac{j}{\omega c_s}}
\]

where, \( r_{pl} \) is the plasma resistance per unit length and \( c_s \) is the sheath’s capacitance per unit length. The plasma resistance is given by:

\[
r_{pl} = \frac{V_\alpha}{|I_{pl}|}
\]

where \( V_\alpha \) is a constant voltage varying with the kind of gas mixture and pressure. From the data reported in [9] it can be taken in the interval 40–70 V. In our calculations we have taken its value as 50 V. The value of \( I_{pl} \) can be obtained from Eq. (6). The sheath’s capacitance in Eq. (9) is given by [7]:

\[
c_s = \frac{\varepsilon_0 A}{d_s}
\]

where \( d_s \) is the sheath’s thickness and “\( A \)” is the area of the electrode. The value of \( d_s \) can be calculated from the invariant law given in [9] as \( fd_s = 42 \), where, “\( f \)” is the excitation frequency. From Eqs. (9)–(11) we obtain:

\[
G_{pl}(x) = j\omega c_s \left[1 - \frac{V_\alpha}{|V(x)|} \exp \left( j \arctan \left( \frac{\text{Im} G_{pl}}{\text{Re} G_{pl}} \right) \right) \right]
\]

Considering that:

\[
I_{pl}(x) = G_{pl}(x)V(x)
\]

Since we have the relation for the power as:

\[
P(x) = V(x)I(x)
\]

The change in temperature \( \Delta T \) due to change in the heat energy \( Q \) is given by:

\[
Q = mC_p(T)\Delta T
\]

where, \( C_p(T) \) is the temperature dependent specific heat capacity at constant pressure for the gas mixture [13, 14] and “\( m \)” is the mass of gas mixture. We also know that \( Q = P(x) t \), where, “\( t \)”
is applied pulse width. From Eqs. (14) and (15) the relation for the temperature distribution is given as:

\[ T(x) = T_0 + \frac{P(x)t}{mC_p(T)} \]  

(16)

where, \( T_0 \) is the room temperature.

The temperature distribution along the longitudinal direction of the waveguide channel is studied theoretically while the gas is pumped through a pulse of pulse length 10 \( \mu \)sec [15]. The ratio of the laser gas mixture is \( \text{CO}_2: \text{N}_2: \text{He}: \text{Xe}=1:1:3:0.25\% \) and the pressure of gas is considered to be as 80 torr. The values for \( L \) and \( C \) used in this analysis are 2.4 nH/cm and 40 pF/cm respectively. Since the resistance of the plasma is not a constant quantity while the discharge is in the running condition so the distribution of plasma resistance is also included.

The current distribution along the longitudinal direction of the electrode system is shown in Fig. 1. It is clear from the figure that the current at the centre is maximum and it decreases exponentially on both sides of the feed point with the increase of position. It is due to the fact that as the input energy is transmitted along the longitudinal direction, the gas molecules are heated up and so the resistance of the gas plasma increases which results in the decrease of the current.

\[ \text{Figure 1: Current distribution along the longitudinal direction.} \]

\[ \text{Figure 2: Voltage distribution along the longitudinal direction.} \]

Figure 2 represents the voltage distribution along the longitudinal direction of the electrode. It can be seen that the voltage increases exponentially toward the ends of the electrodes on both sides of the feed point with the increase of position. The results are in good agreement with the results published elsewhere [2, 7].

\[ \text{Figure 3: Resistance distribution along the longitudinal direction.} \]

\[ \text{Figure 4: Power distribution along the longitudinal direction.} \]

Figure 3 represents the resistance distribution along the longitudinal direction. It can be seen that the resistance increases towards the electrodes ends and is minimum at the central feed point. It is due to the fact that the neutral plasma resistance increases with the increase of the time in pulse mode operation. This is attributed to the different gas composition due to the discharge driven molecular dissociation and consequent formation of new species [2].
Figure 4 represents the distribution of power along the longitudinal direction. It is clear from the figure that it is maximum at the central feed point and decreases exponentially towards the ends of the electrodes. If we consider the Figs. 1, 3 and 4 it becomes clear that the current is maximum at the central feed point, the resistance of the plasma is minimum at the central feed point resulting in the maximum power at the centre. Indeed in the pulsed regime, the dynamics of power transfer to the gas has to be taken into account, producing time-varying density distributions deeply different from those of the CW operation, and often determining pressure waves during the pulse [16]. So the inhomogeneity is to be taken into account not only caused by the locally different electron density but also to the locally different gas density and temperature. Moreover the equilibrium between dissociation and recombination of species could be a source of nonhomogeneous effects considering the relevant fraction of energy transferred to these processes.

Figure 5 represents the temperature distribution along longitudinal direction. It can be seen from the figure that the temperature decreases exponentially towards the ends of the electrode structure. This decrease of the gas temperature in the laser cavity can be explained by considering the direct relationship of power and temperature (Eq. (16)). As the power transferred to the gas molecules is maximum (Fig. 4) at the center and decreases exponentially with the increase of the position from the central feed point so the temperature is maximum at the center and decreases exponentially with the increase of the position on both sides from the central feed point.

REFERENCES


