Modeling the Response of a Seafloor Antenna in the Limits of Low Frequency and Shallow Water

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Abstract — In this paper we describe a novel FDFD (finite-difference frequency-domain) method for computing the 2-D earth response of a 3-D subsea horizontal electric dipole (HED) source. It represents scattered field solutions to Maxwell’s equations and uses a 1-D three-layer analytical solution for the primary field calculations. Comparison with the 1-D industry type of program CSEM1D gave accurate fits.

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1. INTRODUCTION
The basis of the Sea Bed Logging (SBL) approach is the use of a mobile horizontal electric dipole (HED) source and an array of seafloor electric field receivers [3]. The transmitting dipole emits a low frequency electromagnetic signal (fundamental frequency typically ranging from 0.25 to 2 Hz) that diffuses outwards, and the method relies on the large resistivity contrast between hydrocarbon-saturated reservoirs and the surrounding sedimentary layers. The generated electromagnetic soundings can in general be divided into three main contributions: direct EM field, guided modes (associated with high-resistivity zones like hydrocarbons) and air waves (cf. Fig. 2(a)). In case of shallow water depths (e.g., 200 meters or less) the presence of strong airwaves will mask the subsurface signals. In order to develop improved data processing schemes for such cases, modeling tools are needed that can provide controlled test data. In addition, modeling is important when planning new SBL surveys and also when carrying out feasibility analysis of possible new prospects. Since electromagnetic modeling based on time-domain methods give prohibitive large computation times in case of an air layer we concentrate on frequency-domain (methods). Several EM-modeling programs have been developed [1, 5, 7]). However, none of these codes have neither been tailored to the SBL-case nor been documented to work accurately for the case of a low frequency operating seafloor antenna deployed in shallow water.

2. FUNDAMENTALS OF THE MODELING METHOD
We make use of basic ideas employed by Hohmann (1975) [6] to develop a FDFD-method able to compute the earth response of a 3-D sub sea HED-source, assuming a local 2-D earth model (e.g., 2.5-D case). Hohmann introduced scattered field solutions to Maxwell’s equations and proposed to use analytical solutions for the primary field calculations (he considered a uniform background only). By adapting the scattered field approach one avoids the source singularity problem. We employed here a reference model consisting of three layers (i.e., 1.5-D modeling) (Fig. 1). This reference model is solved first, employing analytical solution techniques. The full model is then computed using the reference model as a background solution.

2.1. Primary Field (Reference) Solution
We introduce a Cartesian coordinate system (cf. Fig. 1), and let the x-axis represent the cross-line direction, along which the parameters do not vary. Moreover, we assume a cross-line polarized HED-source. The reference model can be divided into four different solution regions (Fig. 1). The Maxwell’s equations can now be spatially Fourier transformed along both the x- and y-direction.
Based on combinations of these transformed equations, wave-equations for the field components \( \hat{E}_{jx} \) and \( \hat{H}_{jx} \) can be derived (subscript \( j \) representing region \( j \), and the \{∧\}-symbol implies doubly spatially Fourier transformed fields):

\[
\left[ \partial_z^2 + k_{jz}^2 \right] \hat{H}_{jx} = 0 \quad \text{and} \quad \left[ \partial_z^2 + k_{jx}^2 \right] \hat{E}_{jx} = \frac{(k_j^2 - k_x^2)}{i\omega\varepsilon_j^\ast} \delta(z - d), \quad \text{where} \quad k_j^2 = \omega^2\mu_0\varepsilon_j^\ast, \quad \varepsilon_j^\ast = \varepsilon_j + \frac{i\sigma_j}{\omega} \quad (1)
\]
Figure 1: Schematics of the 1.5-D reference model (three layers) including the different solution regions.

where \( I \) is the source current and \( dl \) is its infinitesimal length. Moreover, the source is placed a distance \( d \) above the seafloor. In order to construct feasible solutions a radiation condition must be imposed: only solutions decaying with propagation distance can be allowed within the air and earth half spaces. Hence, a sign-convention for the vertical wavenumber \( k_{jz} \) must be set:

\[
k_{jz} = \sqrt{k_j^2 - k_x^2 - k_y^2} = \sqrt{i \sigma_j \omega \mu_0 + \varepsilon_j \omega^2 \mu_0 - (k_x^2 + k_y^2)} \tag{2}
\]

Inside the water layer (solution regions 2a and 2b in Fig. 1) the complete and general solutions read

\[
\hat{E}_{jx} = \gamma \exp[\mathcal{I}k_{jz} \cdot z] + \eta \exp[-\mathcal{I}k_{jz} \cdot z] - \frac{(k_j^2 - k_y^2) Idl}{2k_{jz} \omega \varepsilon_j^+} \exp[\mathcal{I}k_{jz} \cdot |z - d|],
\]

\[
\hat{H}_{jx} = \alpha \exp[\mathcal{I}k_{jz} \cdot z] + \beta \exp[-\mathcal{I}k_{jz} \cdot z] \tag{3}
\]

where \( \alpha, \beta, \gamma \) and \( \eta \) are complex constants. For solution regions 1 and 3 (cf. Fig. 1) the solutions must be source-free and outward-propagating only. Next, assuming continuity of the tangential components of the fields across the boundaries gives:

### 2.2. Solution region 1

\[
\hat{H}_{1x} = c_1 \exp[\mathcal{I}k_{2z} \cdot z_0 + ik_{1z} (z - z_0)] - 2ic_2 \sin (k_{2z} \cdot z_0) \exp[\mathcal{I}k_{1z} (z - z_0)] \tag{4a}
\]

\[
\hat{E}_{1x} = \omega \mu_0 c_3 \exp[\mathcal{I}k_{2z} \cdot z_0 + ik_{1z} (z - z_0)] - 2ic_4 \omega \mu_0 \sin (k_{2z} \cdot z_0) \exp[\mathcal{I}k_{1z} (z - z_0)]
\]

\[
- \frac{\omega \mu_0 (k_j^2 - k_y^2) Idl}{ik_j^2 k_{2z}} \sin (k_{2z} \cdot d) \exp[\mathcal{I}k_{2z} \cdot z_0 + ik_{1z} (z - z_0)] \tag{4b}
\]

### 2.3. Solution regions 2A and 2B

\[
\hat{H}_{2x} = c_1 \exp[\mathcal{I}k_{2z} \cdot z] - 2ic_2 \sin (k_{2z} \cdot z) \tag{5a}
\]

\[
\hat{E}_{2x} = \omega \mu_0 c_3 \exp[\mathcal{I}k_{2z} \cdot z] - 2i \omega \mu_0 c_4 \sin (k_{2z} \cdot z) - \frac{\omega \mu_0 (k_j^2 - k_y^2) Idl}{ik_j^2 k_{2z}} \sin (k_{2z} \cdot d) \exp[\mathcal{I}k_{1z} \cdot z_0], \ d < z < 0 \tag{5b}
\]

\[
\hat{E}_{2x} = \omega \mu_0 c_3 \exp[\mathcal{I}k_{2z} \cdot z] - 2i \omega \mu_0 c_4 \sin (k_{2z} \cdot z) - \frac{\omega \mu_0 (k_j^2 - k_y^2) Idl}{ik_j^2 k_{2z}} \sin (k_{2z} \cdot z) \exp[\mathcal{I}k_{1z} \cdot d], \ 0 < z < d
\]

### 2.4. Solution region 3

\[
\hat{H}_{3x} = c_1 \exp[-ik_{3z} \cdot z], \ \hat{E}_{3x} = \omega \mu_0 c_3 \exp[-ik_{3z} \cdot z] \tag{6}
\]

Finally, we assume continuity of the \( y \)-component of the \( E \)- and \( H \)-field across the same two interfaces. Hence, we obtain four independent equations giving the solution for the coefficients \( c_1, \)
c_2, c_3 and c_4. The linear system of equations can be written as:

\[
\begin{bmatrix}
a_1 & a_2 & a_3 & a_4 \\
a_3 & a_4 & a_5 & a_6 \\
a_7 & a_8 & a_9 & 0 \\
a_9 & 0 & a_{10} & a_{11}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2 \\
c_3 \\
c_4
\end{bmatrix}
=
\begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4
\end{bmatrix}
\tag{7}
\]

where

\[
a_1 = \left[k_{1z} \left( k_{2z}^2 - k_{1z}^2 \right) - k_{2z} \left( k_{2z}^2 - k_{1z}^2 \right) \right] \exp \left[ i 2 k_{2z} \cdot z_0 \right], \quad a_2 = k_{2z} \left( k_{2z}^2 - k_{1z}^2 \right) + k_{1z} \left( k_{2z}^2 - k_{1z}^2 \right) - a_1 \tag{8a}
\]

\[
a_3 = k_x k_y \left( k_{2y}^2 - k_{1y}^2 \right) \exp \left[ i 2 k_{2z} \cdot z_0 \right], \quad a_4 = k_x k_y \left( k_{2y}^2 - k_{1y}^2 \right) - a_3 \tag{8b}
\]

\[
a_5 = \left[ k_{2x} k_{2z} \left( k_{2y}^2 - k_{2z}^2 \right) - k_1 \left( k_{2y}^2 - k_{2z}^2 \right) \right] \exp [i 2 k_{2z} \cdot z_0], \quad a_6 = \left[ k_{2x} \left( k_{2y}^2 - k_{2z}^2 \right) - k_{1x} \left( k_{2y}^2 - k_{2z}^2 \right) \right] \exp [i 2 k_{2z} \cdot z_0] \tag{8c}
\]

\[
a_7 = k_{3z} \left( k_{2z}^2 - k_{3z}^2 \right) + k_{2z} \left( k_{3z}^2 - k_{2z}^2 \right), \quad a_8 = -2 k_{2z} \left( k_{3z}^2 - k_{2z}^2 \right), \quad a_9 = k_x k_y \left( k_{3z}^2 - k_{2z}^2 \right) \tag{8d}
\]

\[
a_{10} = -k_3^2 k_{3z} \left( k_{2z}^2 - k_{3z}^2 \right) - k_{2z} k_{3z} \left( k_{3z}^2 - k_{2z}^2 \right), \quad a_{11} = 2 k_2^2 k_{2z} \left( k_{3z}^2 - k_{2z}^2 \right) \tag{8e}
\]

\[
b_1 = [idk_x k_y \left( k_{2x}^2 - k_{2z}^2 \right) \sin \left( k_{2z} \cdot d \right) \left( k_{2z}^2 - k_{1z}^2 \right)] \exp \left[ i 2 k_{2z} \cdot z_0 \right] / (ik_{2z} k_{2y}) \tag{8f}
\]

\[
b_2 = -i dk \sin \left( k_{2z} \cdot d \right) \left[ \left( k_{2x}^2 - k_{2z}^2 \right) \left( k_{2y}^2 - k_{2z}^2 \right) - k_{1x} k_{2z}^2 \left( k_{2y}^2 - k_{2z}^2 \right)^2 / (k_{2z} k_{2y}) \right] \exp \left[ i 2 k_{2z} \cdot z_0 \right] \tag{8g}
\]

\[
b_3 = Idk \left( k_{2y}^2 - k_{2z}^2 \right) \left( k_{3z}^2 - k_{2z}^2 \right) \exp \left[ i 2 k_{2z} \cdot d \right] \tag{8h}
\]

Having solved for these two components, the remaining field components are given as (follows from combinations of the doubly spatially Fourier transformed Maxwell’s equations):

\[
\begin{align*}
\tilde{E}_{jy} &= \left[ -k_x k_y \tilde{E}_{jy} + i \omega \mu_0 \partial_z \tilde{H}_{jy} \right] / \left( k_{2z}^2 - k_{2y}^2 \right), \quad \tilde{E}_{jz} = \left[ i k_x \partial_z \tilde{E}_{jy} + k_y \omega \mu_0 \tilde{H}_{jy} \right] / \left( k_{2z}^2 - k_{2y}^2 \right) \tag{9a}
\end{align*}
\]

\[
\begin{align*}
\tilde{H}_{jy} &= \left[ -i \omega \epsilon_0 \partial_z \tilde{E}_{jy} - k_y k_x \tilde{H}_{jy} \right] / \left( k_{2z}^2 - k_{2y}^2 \right), \quad \tilde{H}_{jz} = \left[ -i \omega \epsilon_0 k_y \tilde{E}_{jy} + i k_x \partial_z \tilde{H}_{jy} \right] / \left( k_{2z}^2 - k_{2y}^2 \right) \tag{9b}
\end{align*}
\]

Finally, the primary (reference) fields are found after employing an inverse spatial Fourier transform over the y-coordinate. Up till now, we have considered a HED-source polarized in the cross-line direction. The case of an inline polarized source can be solved in a very similar manner using simple coordinate transformations.

### 2.5. Scattered (Difference) Field Solution

Subtracting the reference model from the general model give the difference fields [7]:

\[
\nabla \times \mathbf{E}^d = i \omega \mu_0 \mathbf{H}^d, \quad \nabla \times \mathbf{H}^d = -i \omega \epsilon^* \mathbf{E}^d - i \omega \Delta \epsilon^* \mathbf{E}^0 \quad \text{where} \quad \Delta \epsilon^* = \epsilon^* - \epsilon^{0*} \approx i \Delta \sigma / \omega \tag{10}
\]

In Eq. (10) the term \(-i \omega \Delta \epsilon^* \mathbf{E}^0\) can be interpreted as a generalized source where \(\mathbf{E}^0\) represents the reference solution of the electric field. Moreover, for a conductive medium \(\Delta \epsilon^*\) can be interpreted as the scaled difference between the total 2-D conductivity and the 1-D background conductivity. Since we assume a 2.5-D formulation it implies that the electric parameters vary in two dimensions only. But the source is a dipole of finite-length, hence a 3-D one. A spatial Fourier transform of Eq. (10) can be carried out along the invariant x-direction. Combination of these transformed equations (scalar versions) give coupled equations for \(\tilde{E}_{jx}^d\) and \(\tilde{H}_{jx}^d\), the difference, along-strike fields in the spatial FT-domain (with \(\gamma^2 = k_{x}^2 - k_{z}^2\)):

\[
\begin{align*}
\partial_y \left[ \sigma \partial_y \tilde{E}_{jx}^d / \gamma^2 \right] + \partial_z \left[ \sigma \partial_z \tilde{E}_{jx}^d / \gamma^2 \right] - \sigma \tilde{E}_{jx}^d &= \Delta \sigma \tilde{E}_{jx}^0 + ik_x \left[ \partial_y \left( \Delta \sigma \tilde{E}_{jy}^0 / \gamma^2 \right) - \partial_z \left( \Delta \sigma \tilde{E}_{jz}^0 / \gamma^2 \right) \right] \\
+ik_x \left[ \partial_y \left( 1/\gamma^2 \right) \partial_z \tilde{H}_{jx}^d - \partial_z \left( 1/\gamma^2 \right) \partial_y \tilde{H}_{jx}^d \right] \\
\partial_y \left[ \partial_y \tilde{H}_{jx}^d / \gamma^2 \right] + \partial_z \left[ \partial_z \tilde{H}_{jx}^d / \gamma^2 \right] - \tilde{H}_{jx}^d &= -\partial_y \left( \Delta \sigma \tilde{E}_{jy}^0 / \gamma^2 \right) + \partial_z \left( \Delta \sigma \tilde{E}_{jz}^0 / \gamma^2 \right) \\
+ik_x \left[ \partial_y \left( 1/\gamma^2 \right) \partial_z \tilde{E}_{jx}^d - \partial_z \left( 1/\gamma^2 \right) \partial_y \tilde{E}_{jx}^d \right] / (\omega \mu_0) \tag{11a}
\end{align*}
\]

The (+-)symbol has been introduced to remind that the field components are spatially Fourier transformed. To compute the reference source, a range of non-zero \(k_x\)-values must be
considered to represent the fields in the FT-domain. In the actual implementation we use a total number of 12 equally spaced (in the logarithmic domain) wavenumbers ranging from 0 to 0.013 m$^{-1}$.

Now assume the along-strike total field components $\tilde{E}_y = \tilde{E}_y^0 + \tilde{E}_y^d$ and $\tilde{H}_z = \tilde{H}_z^0 + \tilde{H}_z^d$ are given. The other components can be calculated by numerical differentiation of these principal components employing the following equations [derived from Eq. (10) after spatial FT]:

$$
\tilde{E}_y = \left[ -i \omega \mu_0 \partial_x \tilde{H}_z^d - i k_x \partial_y \tilde{E}_y^d + i \omega \mu_0 \Delta_\sigma \tilde{E}_y^0 \right] / \gamma^2 + \tilde{E}_y^0,
$$

(12a)

$$
\tilde{E}_z = \left[ -i \omega \mu_0 \partial_y \tilde{H}_z^d - i k_x \partial_x \tilde{E}_y^d + i \omega \mu_0 \Delta_\sigma \tilde{E}_z^0 \right] / \gamma^2 + \tilde{E}_z^0,
$$

(12b)

Equations. (11a) and (11b) are 2-D elliptical partial differential equations. They were solved numerically employing a finite-difference scheme with all first derivatives being represented by centered two-point operators as shown in Eq. (13):

$$
\begin{align*}
& a(i+1/2, j) \cdot \{ U(i+1, j) - U(i, j) \} \frac{1}{\Delta y^2} + a(i-1/2, j) \cdot \{ U(i-1, j) - U(i, j) \} \frac{1}{\Delta y^2} \\
& + a(i, j+1/2) \cdot \{ U(i, j+1) - U(i, j) \} \frac{1}{\Delta x^2} + a(i, j-1/2) \cdot \{ U(i, j-1) - U(i, j) \} \frac{1}{\Delta x^2} \\
& + b(i, j) U(i, j) = s(i, j)
\end{align*}
$$

(13)

where $U$ represents the along-strike electric or magnetic field components, $a$ and $b$ are space-variant and complex coefficients, $\Delta y$ and $\Delta x$ represent grid sizes and $s$ is the discrete representation of the generalized source term. The system in Eq. (13) can be written formally as $M \tilde{u} = \tilde{s}$, where $M$ is a complex matrix containing the known space-variant FD-operators, $\tilde{u}$ is the complex EM-wave vector (to be solved) and $\tilde{s}$ is the known complex source vector. In the actual implementation, $M$ is modified taking into account the boundary conditions. We used here a complex two-point absorbing boundary condition [4]. The matrix $M$ will be a big and sparse matrix of dimension $N \times N$ where $N$ is equal to the total number of grid points. Hence, one needs an efficient way of storing non-zero numbers. Here we chose to use the so called compressed sparse row (CSR) format. The system in Eq. (13) was solved iteratively employing the conjugate gradient method. Prior to this, the system was preconditioned using incomplete LU factorization (ILU), and transformed to its normal-equation form:

$$
\hat{M}^T \hat{M} \hat{u} = \hat{M}^T \hat{s} \ \text{where} \ \hat{M} = K^{-1} M, \ \hat{s} = K^{-1} \tilde{s}
$$

(14)

Figure 2: (a) Schematics of test model, (b) Relative magnitude and (c) residual phase of inline polarized HED-source (e.g., with and without hydrocarbon layer). Maximum source-receiver offset was 8 km. Comparison with CSEM1D-program (solid curve).
where $T$ means taking the matrix transpose the asterix denotes complex conjugating and $K$ is the preconditioning matrix. The system matrix $\hat{M}^T \hat{M}$ in Eq. (14) is Hermitian. In such a case it can be shown that an iterative formulation of Eq. (14) based on the conjugate gradient method will be stable [8]. The coupled Eqs. (11a) and (11b) are solved in an alternating fashion by including the coupling terms on the right-hand sides of the equations in the source term.

3. VALIDITY OF THE METHOD

There is a general lack of analytical expressions for 2-D models, so our method was tested against the program CSEM1D, which calculates the response of a Hertz-dipole source in the frequency-domain for a 1-D model. The CSEM1D-program is based on the theory described by Chave and Cox (1982) [2]. Fig. 2(a) shows a sketch of the test model. The HED-source was placed 50 m above the seafloor and had an operating frequency of 0.25 Hz. A water depth of 200 m guaranteed a significant airwave contribution. The conductivities of the overburden and the 200 m thick oil layer were 1 Ω-m and 50 Ω-m, respectively.

Field components were computed for an inline polarized source and receivers placed on the seafloor. Our code showed good agreement with the CSEM1D-program (cf. Figs. 2(b) and (c)), where we have computed the relative (horizontal) electric field, e.g., the ratio between the $E_y$ response associated with the hydrocarbon layer and a homogeneous subsurface, respectively.

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