Analysis of Evanescent Waves Scattering by a Single Particle in Total Internal Reflection Microscopy

E. Eremina$^1$, T. Wriedt$^1$, and L. Helden$^2$

$^1$University of Bremen, Germany
$^2$University of Stuttgart, Germany

Abstract—Since its invention in the mid of eighties [1] Total Internal Reflection Microscopy (TIRM) has proven to be an effective technique to measure weak interactions between spherical colloidal particles and surfaces with a resolution of a few femtonewton. It is a single particle evanescent light scattering technique. In an experimental setup a laser beam is coupled into a prism and hits the glass-water interface with an angle slightly above the critical angle of total internal reflection. This generates an evanescent field near the interface that decays in the lower refractive index medium (water) with a characteristic penetration depth which depends on the angle of incidence. A colloidal particle that is dispersed in the medium will scatter light from the evanescent wave if it is in the vicinity of the surface. By registering a scattered intensity it is possible to deduce the particle-substrate distance. Compared to other methods for measure particle wall interactions like the surface force apparatus or the atomic force microscopy where a colloidal particle is attached to the tip, TIRM is the most sensitive technique because thermal fluctuations where limit the other methods in their resolution are exploited.

To compare experimental results with results of mathematical modeling an effective light scattering method is needed. For this purpose the Discrete Sources Method (DSM) has been chosen. The DSM is a well-known method for light scattering analysis, which has recently been applied for evanescent wave scattering [4].

1. Discrete Sources Method

For the theoretical modeling the Discrete Sources Method (DSM) has been chosen. The DSM is a well-known method for the analysis of light scattering. It has recently been applied to the evanescent wave scattering [4]. In frame of the DSM the mathematical statement can be presented as follows:

$$\nabla \times \mathbf{H}_\zeta = jk \varepsilon_\zeta \mathbf{E}_\zeta; \quad \nabla \times \mathbf{E}_\zeta = -j \mu_\zeta \mathbf{H}_\zeta \quad \text{in} \ D_\zeta, \ \zeta = 0, 1, i,$$

$$\mathbf{n} \times (\mathbf{E}_i - \mathbf{E}_0) = 0, \ \mathbf{n} \times (\mathbf{H}_i - \mathbf{H}_0) = 0, \ \text{at} \ \partial D$$

$$\mathbf{e}_z \times (\mathbf{E}_0 - \mathbf{E}_1) = 0, \ \mathbf{e}_z \times (\mathbf{H}_0 - \mathbf{H}_1) = 0, \ \text{at} \ \Sigma$$

(1)

and radiation conditions at infinity.

Here, $D_0$ is an ambient media, $D_1$ is a glass prism, $D_i$ is an interior particle domain $\partial D$ is a particle boundary, $\Sigma$ is a prism-air border, $\mathbf{n}$ is the outward unit normal vector to $\partial D$, $k = \omega / c$ and $\{\mathbf{E}_\zeta, \mathbf{H}_\zeta\}$ stands for the total field in the corresponding domain$D_\zeta$. We assume that the exciting field $\{\mathbf{E}_1, \mathbf{H}_1\}$ is a plane wave propagating from $D_1$ at the angle $\theta_1$ with respect to the z-axis and transmitting at the interface following Snell’s law. Note that the total field in $D_0$ is a superposition of the refracted incident field $\{\mathbf{E}_0, \mathbf{H}_0\}$ and the scattered $\{\mathbf{E}_s, \mathbf{H}_s\}$ field. If $\text{Im} \ \varepsilon_\zeta, \mu_\zeta \leq 0$ (the time dependence for the fields is chosen as $\exp\{j \omega t\}$) and the particle surface is smooth enough $\partial D \subset C^{1, \alpha}$, then the above boundary-value problem is uniquely solvable.

We construct an approximate solution to the scattering problem (1) according to the DSM outlines [5]. The amplitudes of discrete sources are determined from the boundary conditions at the particle surface, which can be rewritten as

$$\mathbf{n} \times (\mathbf{E}_i - \mathbf{E}_0) = \mathbf{n} \times \mathbf{E}_0^i \quad \mathbf{n} \times (\mathbf{H}_i - \mathbf{H}_0) = \mathbf{n} \times \mathbf{H}_0^i, \ \text{at} \ \partial D$$

(2)

To construct the fields of dipoles and multipoles analytically satisfying the transmission conditions at the plane interface $\Sigma$ we apply the Green tensor for a stratified interface [6]. An approximate solution takes into account an axial symmetry of the particle and the polarization of an external excitation. For $P$-polarized excitation for fields presentations outside the particle the following electric
and magnetic vector potentials are used

\[ A_{m,n}^{\epsilon,0} = \{ g_m^\epsilon (\eta, z_n) \cos (m + 1) \phi; -g_m^\epsilon (\eta, z_n) \sin (m + 1) \phi; 0 \}, \]

\[ A_{m,n}^{h,0} = \{ g_m^h (\eta, z_n) \sin (m + 1) \phi; g_m^h (\eta, z_n) \cos (m + 1) \phi; 0 \}, \]

\[ A_{m,n}^{\epsilon,h} = \{ 0; 0; g_m^\epsilon, h (\eta, z_n) \}. \]  \hspace{1cm} (3)

where \( g_m^\epsilon, h \), \( f_m \) Fourier harmonics of the corresponding Green tensor components [4]. For the total field inside the particle we define the following vector potentials:

\[ A_{m,n}^{\epsilon,i} = \{ J_m^i (\eta, z_n) \cos (m + 1) \phi; -J_m^i (\eta, z_n) \sin (m + 1) \phi; 0 \}, \]

\[ A_{m,n}^{h,i} = \{ J_m^i (\eta, z_n) \sin (m + 1) \phi; J_m^i (\eta, z_n) \cos (m + 1) \phi; 0 \}, \]

\[ A_{0,n}^{\epsilon,h} = \{ 0; 0; J_0^i (\eta, z_n) \}. \]  \hspace{1cm} (4)

Here \( J_m^i (\eta, z_n) = j_m (k, R_{qa}) (\rho/R_{qa})^m, j_m (\cdot) \) is the cylindrical Hankel function, \( \{ z_n \}_{n=1}^\infty \) is a dense set of the points distributed over a segment \( \Gamma_s \subset D_1 \) of the axis of symmetry. The approximate solution for the P-polarized excitation can be represented as

\[
\left( \frac{E_n}{H_n^\times} \right) = \sum_{m=0}^M \sum_{n=1}^{N_m} \left\{ p_{m,n}^\epsilon D_1^i A_{m,n}^{\epsilon,c} + p_{m,n}^h D_2^i A_{m,n}^{h,c} \right\} + \sum_{n=1}^{N_0} r_n^\epsilon D_1^0 A_{0,n}^{\epsilon,c}.
\]  \hspace{1cm} (5)

Where \( D_1^i = \left( \frac{j}{\kappa_c \rho} \nabla \times \nabla \times - \frac{1}{\rho_c} \nabla \times \right) \), \( D_2^i = \left( \frac{1}{\kappa_c \rho_c} \nabla \times \frac{j}{\kappa_c \rho_c} \nabla \times \nabla \times \right) \).

The case of an S-polarized excitation can be considered in a similar way [4].

Now we would like briefly describe the numerical realization of the computational algorithm. As mentioned above representation (5) satisfies all the conditions of the scattering problem (1) except the transmission conditions at the particle surface (2). These conditions are used to determine the unknown amplitudes of discrete sources \( \{ p_{m,n}^\epsilon, h; q_{m,n}^\epsilon, h \} \). Since the scattering problem geometry is axially symmetric with respect to the Z-axis and discrete sources are distributed over the axis of symmetry, fulfilling the transmission conditions (2) at surface \( \partial D \) can be reduced to a sequential solution of the transmission problems for the Fourier harmonics of the fields. So, instead of matching the fields on the scattering surface, we can match their Fourier harmonics separately thus reducing the approximation problem to the surface to a set of problems enforced at the particle surface generatrix \( S \). By solving these problems one can determine the discrete sources amplitudes \( \{ p_{m,n}^\epsilon, h; q_{m,n}^\epsilon, h \} \).

For the determination of amplitudes the generalized point-matching technique is used [7]. The DSM is a direct method and hence it allows to solve the scattering problem for the entire set of incident angles \( \theta_1 \) and for both polarizations (P and S) at the same time. Besides, the numerical scheme provides an opportunity to control the convergence of the approximate solution to the exact one by a posterior error estimation [5].

After the amplitudes of the discrete sources (DS) are determined, the far field pattern \( E_\infty (\theta, \varphi) \) of the scattered field, can be calculated. It is determined at the upper part of the unit semi-sphere \( \Omega = \{ 0^\circ \leq \theta < 90^\circ, 0^\circ \leq \phi \leq 360^\circ \} \) and is given by

\[ E_\infty (r) = \exp \left( \frac{j k \rho r}{r} \right) F (\theta, \varphi) + O (r^{-2}) \quad r \to \infty \]

Using asymptotical estimation of the Weyl-Sommerfeld integrals of the Green’s tensor components, the representation of the elements of a far field pattern gets a form of finite linear combinations of elementary functions [4]. This circumstance ensures an economical computer analysis of the scattering characteristics in the wave zone.

One of the most important scattering characteristics is an intensity of scattered light

\[ I^{P,S} (\theta_0, \theta, \varphi) = \left| F_{\theta}^{P,S} (\theta_0, \theta, \varphi) \right|^2 + \left| F_{\varphi}^{P,S} (\theta_0, \theta, \varphi) \right|^2 \]  \hspace{1cm} (6)

where \( F_{\theta, \varphi}^{P,S} (\theta_0, \theta, \varphi) \) are the components of the far field pattern for P (5) and S polarized incident wave, in a spherical coordinate system \( \theta, \varphi \). In the paper we will examine the objective response function, which presents the intensity scattered into a certain solid angle:

\[ \sigma_s^{P,S} (\theta_0) = \int I^{P,S} (\theta_0, \theta, \varphi) \, d\omega \]  \hspace{1cm} (7)
where $\Omega = \{ 0 \leq \varphi \leq 360^\circ; 0 \leq \theta \leq \theta_{NA} \}$, $\theta_{NA}$ is an angle, which corresponds to the Numerical Aperture (NA) of the objective lens in accordance with $\theta_{NA} = \arcsin (NA/n_0)$.

The number of matching points where the DS amplitudes are defined increases till the necessary accuracy of the results is achieved. The DS number usually is 2-3 times less then the number of the matching points on the particle generatrix. As a rule the discrete sources are deposited on the axis of symmetry inside the particle. The order of multipoles (M) is a priori defined from the condition that the plane wave approximation should be less then 0.1%. The detailed algorithm of matching point’s choice and deposition is described in [6].

2. Results and Discussion

In the paper we would like to present some results of numerical modelling of the TIRM calibration curve. As an example we took a PSL sphere of diameter $D = 1.6 \, \mu m$ at wavelength of 658nm. In figures the objective response (7) is plotted as a function of particle–prism distance. To show the results we have chosen two incident angles which differ from critical one. In Figure one the results for PP, PS, SP and SS polarization are presented for deviation of the incident angle from critical one by 0.58°, and in figure 2 similar results are presented for deviation of 2.6°. In both figures the intensity for P polarized light has less distortions then for S polarized. It is in good agreement with a multiple reflections theory, as the reflectance for P-polarized light is always lower then for S-polarized one.

![Figure 1](image1)

**Figure 1:** The objective response (7) versus particle-prism distance for a PSL sphere of diameter $D = 1.6 \, \mu m$ for different polarizations and incident angle deviation of 0.58°.

![Figure 2](image2)

**Figure 2:** The objective response (7) versus particle-prism distance for a PSL sphere of diameter $D = 1.6 \, \mu m$ for different polarizations and incident angle deviation of 2.6°.

In the oral presentation more numerical results and their comparison with experimental data will be shown.

Acknowledgement

Authors would like to acknowledge financial support of this work by Deutsche Forschungsgemeinschaft (DFG).

REFERENCES