Higher Order Fourier Analysis for Multiple Species Plasma

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Abstract—Several first order perturbations of moderate amplitude may easily occur together in nature in a small interval of time. Each separately leads to a family of higher order terms which may total a somewhat larger amplitude. However, all the nonlinear terms of all first order perturbations may lead for a certain phase to a very large and even a divergent result. A kind of bunching, concentration of the energy in a small phase interval, occurs. This may act as a trigger and explain sudden outbursts which occur in nature (e.g., in solar flares, CMEs, bright points, prominences, etc.), on Earth (interruption of power generators) and in the laboratory. Inducing several moderate perturbations in a quiet plasma (e.g., a Q-machine) may allow experimental verification of the theoretical convergence limit.

1. Introduction

The nonlinear Fourier method of Callebaut consists in concentrating on the family of higher order terms of a single Fourier term of the linearized analysis [1–3]. Thus we have obtained the higher order terms of plasma perturbations, gravitational ones, etc. In the simplest case of a cold plasma this resulted in obtaining an analytical expression for the higher order terms. This allowed to investigate the convergence of the series, which in this case is $e^{-1}$ of the equilibrium density. For the cases without an analytical expression we developed a numerical-graphical method to obtain the convergence limit. Near this limit the total amplitude of the wave becomes very large. The convergence limit decreases with increasing pressure.

We made an attempt [5] to explain a baffling aspect of some plasmas, e.g., prominences, that remain quiet during weeks, even months, and then suddenly burst out in an explosion with no apparent reason. Now we add a powerful argument to the reasoning in [5]: several first order perturbations of moderate amplitude may easily occur together in nature in a small interval of time. Each separately leads to a family of higher order terms which may total somewhat larger amplitudes. However, all the nonlinear combination of all first order perturbations may lead for a certain phase to a very large and even a non-convergent result.

2. A Basic Result

The nonlinear Fourier analysis by Callebaut was developed in several papers [1–3]. In particular an analytic expression for the cold plasma case was developed [2]. Consider a first order perturbation $n_1$ of the density with amplitude $A$: $n/n_0 = Ae^{i(\omega t + kx)}$ normalized to the equilibrium density, using conventional notations. The associated family for the density) reads

$$n/n_0 = \sum_{s=1}^{\infty} \frac{s^s}{s!} A^s e^{is(\omega t + kx)}.$$  \hspace{1cm} (1)

Similar expressions are valid for the velocity, the potential and the pressure.

This analytic expression allows to calculate the convergence ratio: $A < e^{-1}$. This means that the amplitude of the first order term has to be less than 37 per cent of the equilibrium density. Otherwise the associated sum, i.e., the full perturbation, diverges for at least one phase.

3. Several First Order Terms

Two remarks are here in order. The first one is that the above analysis is in fact an ordinary Fourier analysis of a specific solution of the system of equations with specific initial conditions: $A, \omega, k$ (which are in fact given by the first order term). In that sense the Fourier analysis is mathematically safe, as long as some very general conditions are satisfied of which the main one is that the situation is periodic, which is the case for the cold plasma. Nonlinearity comes in only when considering more than one of such first order terms. In that case interference or mixing occurs between the various families. However, once the series (1) is known, the solution for the nonlinear situation follows immediately from the binomial theorem:
\[
\frac{n}{n_0} = 1 + \sum_{s=1}^{\infty} \frac{s^s}{s!} (A e^{i(\omega t + kx)} + B e^{i(\mu \omega t + kx)} + C e^{i(\lambda \omega + kx)})^s.
\]

\(B\) and \(C\) are first order amplitudes like \(A\). The factors \(\mu\) and \(\lambda\) are introduced as the various perturbations have various frequencies; for simplicity we kept the same wavenumber \(k\). The extension to any number of first order terms is obvious.

The second remark concerns the initial amplitude required to have a divergent series: for the case of a single first order term this amplitude has to exceed 37 per cent of the equilibrium density to yield an instability. This requires usually an extremely strong perturbation, only possible if an extraordinary (big) bang is applied to the plasma. The limit is lowered when pressure terms are included, but even 5 or 10 per cent of the equilibrium are strong first order perturbations. This was a weak point in our paper (5). In nature or in the laboratory you need practically a neighboring explosion or the application of a (sudden) very large perturbing field. Ordinary shocks may not be sufficient. However, the application is quite different when we consider several perturbations as illustrated in equation (2). Now it is the sum of \(A + B + C + ...\) that has to reach 0.37 to have a diverging total sum for some phases. If in nature e.g., perturbation occur with amplitudes around 0.01 then some 37 of such perturbations may blow up the plasma once the phase is reached where the sum becomes divergent. This is not an impossible demand, especially as the perturbations do not have to be originally in phase or have their frequencies in a rational proportion. This may allow quite a delay before the instability sets in, a feature which suits many observations. See below: applications.

We have drawn figures for several values of the parameters \(A, B, C, \mu, \lambda\): \(A + B + C\) varied from 0.05 to 0.4, \(\mu\) and \(\lambda\) took values 2, 5, \(\sqrt{2}\), \(\sqrt{10}\). We considered various orders: \(s = 1, 3, 10, 100\). Here we put some conclusions

1. If the total sum of the amplitudes is smaller than about 0.2 then the difference between order 3, 10, and 100 is very small.
2. If the total sum of the amplitudes is greater than 0.2 but less than about 0.3 then the difference between order 10 and 100 is very small.
3. If the total sum of the amplitudes is near 0.35, then one sees some difference between order 10 and 100.
4. For a total sum of the amplitudes above 0.35 the difference increases more and more. Above 0.37 (or \(e^{-1}\)) the series diverges (except for very special cases like the one in the cosine development). Moreover, the total density becomes negative (total density becomes less than zero for some arguments, which is physically impossible); this confirms that the series is not convergent.
5. The calculation time (using pentium 4 computer) is reasonable up to order 100. However, the time increases for each wave which is added especially for high orders.
6. Starting from several small amplitudes one may reach a solitonlike behavior for a certain phase (either in time or in space) with a quite large amplitude, especially when the sum of the initial amplitudes approaches 37% of the initial density. As in nature several (many) small perturbations may occur more or less together, although they may be generated at different places, this phenomenon may be important.

4. Applications

We just mention a few aspects. We refer to our paper (9), of course now having in mind several perturbations of reasonable amplitudes instead of the very big one required there.

4.1. Solar Flares, Filament Bands, Bright Points and CMEs

E.g., for a solar flare to be initiated waves may come from all sides at all times and it may take a long time (days, weeks) before the limiting value for instability is reached. Moreover, the strip in which the instability is initiated is very narrow (a finite energy is bunched together into a narrow space). From that strip the instability may spread over the whole flux tube (e.g., anomalous resistivity may occur) and thus some time elapses between the ignition and the flash (typically a quarter of an hour) liberating a tremendous amount of energy from a magnetic flux tube.

The same applies to e.g., a so-called “bright point” near the solar surface. Note that several small perturbations may ignite a bright point and that several bright points may ignite a prominence.

4.2. Power Generators on Earth

It is well known that power generators may crack down due to some perturbation which was apparently too small to cause the instability. Cf. March 1989 when the whole state of Quebec, Canada, was a day without electricity due to a solar storm which caused a magnetic perturbation spreading to the Earth. Again those
perturbations seem often too small to have such an effect, but adding all the higher order terms and all the various perturbations may yield instability.

4.3. Ball Lightning

The most baffling feature of ball lightning is that it involves one or two orders of magnitude more energy than what may be expected from its light emission; the corresponding stability is equally surprising, sometimes followed by a quiet evanescent phenomenon, sometimes followed by a strong explosion. In [5] we have attempted to explain the huge energy contained in ball lightning by waves of the type of equation (1), rotating (or moving back and forth), but peaked in a narrow phase band. Moreover the cases where an explosion happens may be due to the combination of several perturbations of moderate amplitude as explained above.

4.4. Experimental Suggestion

In some experimental setups like the Q-machine one obtains a quiet plasma during reasonable periods. One may attempt to reach the instability limit by applying a very strong (sudden) external field, as suggested in [2]. However it may be easier to apply several perturbations (e.g., of different frequencies) each requiring a smaller amplitude. Plasmas with heavy negatively charged ions (fullerenes attach electrons) and positive ions may be suitable for this as their frequencies are much lower than for ordinary plasmas. However, a snag is that some electrons do not attach, resulting in a three species plasma. We are developing the appropriate extension of our nonlinear theory for this multiple species plasma.

5. Conclusion

In [5] we argued that a first order term may have a whole family of associated higher order terms which for some phase all combine together to form a powerful wave which may act as a trigger causing instability. The weak point was that the first order term had to be already very strong. Now, we have shown that the combination of several moderate first order terms and their families can yield a very strong (even divergent) perturbation in a narrow phase strip, thus having the possibility to act as a trigger locally or in a neighboring configuration.

REFERENCES