Optimization of Fourier Plane Coverage of Antenna Arrays for SPORT

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Abstract

The fidelity of image reconstruction of SPORT depends on samplings in the Fourier transform domain of the scene performed by its antennas array. Research in this paper aims at finding the optimal layout of the receiving elements which is fitting for SPORT. Simulated annealing is applied to finding solutions for $N = 8, 10, 12, 14$. The simulation results show that centers of gravity of optimized antenna arrays are situated on center of the circle. Spatial frequency domain is covered evenly by the sampling points after rotation of the optimized antenna array. Moreover, the redundancy of baselines is minimized. The length values of the baselines of the optimized antenna array are also evenly distributed on the interval $[0, 1]$.

Introduction

SPORT (Solar Polar Orbit Radio Telescope) project is a solar wind monitoring mission proposed by Center for Space Science and Applied Research, Chinese Academy of Sciences. Large antenna aperture is required in order to obtain adequate samplings of the spatial frequency domain for SPORT. The interferometric technique is an attractive means of synthesizing large antenna aperture and improving angular resolution for microwave radiometer [1 \textsuperscript{\textendash} 3]. It is one of SPORT’s characteristics to adopt this technique.

The antenna aperture of SPORT is formed by a number of antenna elements. The distribution of these antenna elements determines the samplings of radiation brightness temperature of the scene in the Fourier transform domain. Therefore, the choice of the distribution of the antenna elements is very important for SPORT.

The methodology used to design the element distribution for SPORT is based upon simulated annealing. It is a good technique for solving difficult combinatorial optimization problems [4, 5] and a suitable method for optimization of the Fourier plane coverage of antenna arrays [6 \textsuperscript{\textendash} 8]. However, the optimum results in Ref. [6], [7] and [8] either have excessive redundant baselines or can not ensure superposition of center of gravity and geometrical center of the antenna arrays.

In the next section, design requirements of antenna arrays for SPORT are summarized. In the subsequent section, the fundamental theory of simulated annealing is introduced and its application to optimization of antenna elements distribution for SPORT is discussed. Finally, the optimized results for $N = 8, 10, 12, 14$ are given via numerical simulation.

Design Requirements of SPORT

In order to reduce the number of antenna elements, SPORT mission adopt a time-shared sampling scheme. In this scheme, a thinned array rotates about its center of gravity to take a complete image. Therefore, the phase or direction of the baseline is not what we are interested. In the following discussions, we will only concentrate on the lengths of the baselines by selecting optimal distributions of each element. The design requirements are as follows:

- The antenna elements are situated at a circle with a unit diameter, i.e. the aperture is normalized to the longest baseline.
- The center of gravity of the array is located in the geometrical center of it.
- In order to obtain the complete information of the scene, the spatial frequency domain should be covered as evenly as possible by the sampling points after rotation of the array. The length values of the baselines should also be evenly distributed on the interval $[0, 1]$.
- The redundancy of baselines should be minimized.
Optimization Strategy

The conventional SA algorithm is briefly described as follows [9]: in each step of the algorithm, a random solution in the neighborhood of the current solution is generated and the resulting change in fitness of objective function, $\Delta f$, is computed. If $\Delta f \leq 0$, then the modification is accepted. The case of $\Delta f > 0$ is treated probabilistically. The probability that the modification is accepted is given by Eq.(1):

$$P(\Delta f) = \exp[-\Delta f / T]$$  (1)

where $T$ is the current temperature. A random number, uniformly distributed on the interval $[0, 1]$, is selected and compared with $P(\Delta f)$, then a new assignment is accepted. Thereafter the temperature is lowered according to the annealing schedule. This procedure is repeated until the state of the assignment completely freezes or the termination criterion is fulfilled.

In this paper, the objective function $\Delta f$ is to be identified with the error function $\Delta E$, which is given by Eq.(2):

$$\Delta E = e_{\text{max}}(n; r_1, r_2, \ldots, r_N) - e_{\text{max}}(n - 1; r_1, r_2, \ldots, r_N)$$  (2)

where $r_i$ is the $i$th baseline length of the circular arrays, $e_{\text{max}}$ is

$$e_{\text{max}} = \max([r_1, r_2, \ldots, r_L] - [r_{1-\text{ideal}}, r_{2-\text{ideal}}, \ldots, r_{L-\text{ideal}}])$$  (3)

where $[r_{i-\text{ideal}}]$ is a matrix which consist of the ideal length values of baselines.

Furthermore, another function $\text{num}$, which is the number of baselines, is taken into account.

The appropriate “cooling schedule” of simulated annealing governs the convergence of the algorithm [10]. The parameters of the cooling schedule are [11]: initial temperature $T_0$, cooling rate $\alpha$, terminate temperature $T_{\text{end}}$, and a finite length of each homogenous Markov chain $L_k$. In this paper, we choose $T_0 = 900$, $\alpha = 0.9995$, $T_{\text{end}} = 1$ and $L_k = 100N$ via a great deal numerical simulation.

The major implementation steps of optimizing the distribution of the antenna elements with SA are summarized as follows:

Step (0):
- Initialize $T_0$, $\alpha$, $T_{\text{end}}$ and $L_k$.
- Set iteration counter $s = 1$ and the maximum number of iterations $s_{\text{end}} = 5 \times 10^5$.
- Set initial distribution of receiving elements.
- Calculate $\text{num}$, $e_{\text{max}}$ and the objective function $\Delta E$.

Step (1):
- Generate a new trail distribution.
- Calculate $\Delta \text{num} = \text{num}(n) - \text{num}(n - 1)$.

Step (2):
- If $\Delta \text{num} > 0$, then calculate $\Delta E = e_{\text{max}}(n) - e_{\text{max}}(n - 1)$.
- If $\Delta E < 0$, accept the trail distribution. Else, decrease the temperature according to the annealing schedule and perform the SA acceptance test.

Else, reject the trail distribution and go to step (1).

Step (3):
- If $\text{num}(n)$, $e_{\text{max}}(n)$ satisfy the design requirements then stop, else go to step (1).

Step (4):
- If $s_{\text{end}}$ or $T_{\text{end}}$ is reached then stop, else go to step (1).
Optimization Results

In this paper, by using simulated annealing, solutions are obtained for $N = 8, 10, 12, 14$. The array elements are constrained to locate on a circle which radius is 0.5. An evenly distribution of the array elements on the circle is the simplest condition which can ensure superposition of center of gravity and geometrical center of the antenna arrays. Therefore, they are chose as the initial position angles (in degrees), which are as shown in Table 1. The optimized locations of array elements on the circle are as shown in Table 2. The spatial frequency spectrum and baselines of the optimized array elements are compared with those of the initial array elements, which are as shown in Fig. 1.

It can be seen in Table 1, 2 that centers of gravity of either initial antenna arrays or optimized antenna arrays are situated on center of the circle. All antenna arrays have beautiful crystalline structure, with bilateral symmetry. By virtue of Fig. 1, the comparison of baselines and spatial frequency spectrums of the optimized antenna arrays and of the initial antenna arrays are presented as follows:

a) The number of baselines of initial antenna arrays is 10, 12, 17, 37 for $N = 8, 10, 12, 14$, respectively.

Figure 1: Solutions for $N = 8, 10, 12, 14$ are plotted in diagrams. Baselines and sampling points after optimizing are compared with those before optimizing. Scale is normalized.
Table 1: Initial position angles of the array elements for $N = 8, 10, 12, 14$

<table>
<thead>
<tr>
<th>$N$</th>
<th>0.0</th>
<th>45.0</th>
<th>90.0</th>
<th>135.0</th>
<th>180.0</th>
<th>225.0</th>
<th>270.0</th>
<th>315.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.0</td>
<td>36.0</td>
<td>72.0</td>
<td>108.0</td>
<td>144.0</td>
<td>180.0</td>
<td>216.0</td>
<td>252.0</td>
</tr>
<tr>
<td>10</td>
<td>0.0</td>
<td>30.0</td>
<td>60.0</td>
<td>90.0</td>
<td>120.0</td>
<td>150.0</td>
<td>180.0</td>
<td>210.0</td>
</tr>
<tr>
<td>12</td>
<td>0.0</td>
<td>25.7</td>
<td>51.4</td>
<td>77.1</td>
<td>102.8</td>
<td>150.0</td>
<td>180.0</td>
<td>210.0</td>
</tr>
<tr>
<td>14</td>
<td>0.0</td>
<td>22.5</td>
<td>45.0</td>
<td>67.5</td>
<td>90.0</td>
<td>122.5</td>
<td>155.0</td>
<td>187.5</td>
</tr>
</tbody>
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Table 2: Optimized position angles of the array elements for $N = 8, 10, 12, 14$

<table>
<thead>
<tr>
<th>$N$</th>
<th>0.00</th>
<th>14.86</th>
<th>113.3</th>
<th>140.2</th>
<th>180.0</th>
<th>194.8</th>
<th>293.3</th>
<th>320.13</th>
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<tbody>
<tr>
<td>8</td>
<td>0.00</td>
<td>28.41</td>
<td>105.85</td>
<td>125.3</td>
<td>170.5</td>
<td>180.0</td>
<td>208.4</td>
<td>285.85</td>
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<tr>
<td>10</td>
<td>0.00</td>
<td>3.50</td>
<td>21.94</td>
<td>33.22</td>
<td>88.23</td>
<td>138.5</td>
<td>213.22</td>
<td>268.23</td>
</tr>
<tr>
<td>12</td>
<td>0.00</td>
<td>38.72</td>
<td>48.89</td>
<td>54.59</td>
<td>82.44</td>
<td>153.6</td>
<td>218.72</td>
<td>228.89</td>
</tr>
<tr>
<td>14</td>
<td>0.00</td>
<td>35.38</td>
<td>42.85</td>
<td>49.56</td>
<td>82.44</td>
<td>153.6</td>
<td>218.72</td>
<td>228.89</td>
</tr>
</tbody>
</table>

Correspondingly, the redundancy of baselines is 65.5%, 73.9%, 74.6%, 59.8%, respectively. The length values of baselines are not evenly distributed on the interval [0, 1]. Moreover, although sampling points in spatial frequency domain of initial element distributions are relatively even, the intervals of sampling points with various radiuses are relatively distant after rotation of the array. Consequently, high spatial resolution of the scene cannot be obtained.

b) The number of baselines of the optimized antenna arrays is 25, 41, 57, 78 for $N = 8, 10, 12, 14$, respectively. Correspondingly, the redundancy of baselines is 13.8%, 10.9%, 14.9%, 15.2%, respectively. The reduction of redundancy is 51.7%, 63.0%, 59.7%, 44.6%, respectively. The simulation results also show that, the length values of the baselines of each optimized antenna array are also evenly distributed on the interval [0, 1]. Moreover, it can be seen that spatial frequency domain is covered evenly by sampling points with various radiuses of optimized antenna arrays after rotation of the arrays. Moreover, the intervals of sampling points with various radiuses are relatively close after rotation of the array. Consequently, high spatial resolution of the scene can be obtained.

Conclusion

In this paper, simulated annealing is used for optimization of the antenna elements distribution. Numerical simulation is performed on $N = 8, 10, 12, 14$. It is observed from the simulating results that all optimized antenna arrays have beautiful crystalline structure, with bilateral symmetry. The length values of the baselines of each optimized antenna array are also evenly distributed on the interval [0, 1]. Spatial frequency domain is covered evenly by sampling points with various radiuses after rotation of the arrays. Design requirements of SPORT are satisfied.

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REFERENCES


