Image Simulator for One Dimensional Synthetic Aperture Microwave Radiometer

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Abstract

Calibration for a one-dimensional synthetic aperture microwave radiometer is critical. A numerical image retrieving method using an intermediate transformation matrix called the gain matrix or simply G-matrix is considered to be a good method. This matrix takes into account all the couplings and un-uniformed amplitude and phase between the channels. However, measuring the G-matrix is not an easy job. One has to set up a point source in the far field and rotate the system in the fan beam direction. The difficulty is mainly due to the incomplete test environment.

In this paper, a measurement set-up called image simulator is present. It is a circuit network composed of a number of high accurate microwave I-Q vector modulators and a noise generator. It simulates the incoming waves to the thinned antenna array corresponding to any image scene to be measured.

The design and test of the image simulator are presented followed with test results of G-matrix. The image simulator can also simulate a point source in the two dimensional aperture case which discussed in the paper too.

Introduction

Synthetic aperture microwave radiometer represents a new technology being developed for passive microwave remote sensing [1][2]. Aperture synthesis uses interferometry technique to improve the limitations set by antenna physical aperture while working at lower frequency. Conventional fan-beam phased array antennas form the real aperture. Spatial resolution of the instrument is determined by the size of the antenna beam which is inverse to the aperture of the antenna. Therefore, the size of the antenna should be huge to obtain the desired resolution. Aperture synthesis provides an alternative to conventional radiometers because it enables high spatial resolution at the same time minimizes the antenna size and complexity.

One-dimensional synthetic aperture microwave radiometer uses aperture synthesis across track and employs real antenna aperture along track to attain two-dimensional images of brightness temperature. In the cross track direction, it uses thinned arrays of antennas to form necessary pairs of interferometers. The data produced by the instrument can be separated into the sine and cosine components known as visibility functions and is a measure of a spatial harmonic in the brightness temperature scene.

Differ from the conventional radiometer, synthetic aperture microwave radiometer measures a complete map of brightness temperature in a single snapshot. Thus, perfect calibration is theoretically not impossible without the measurement of a known scene. Measuring the G matrix of the instrument need to measure the response to the noise source with respect to the direction of incident radiation. The experiment should be conducted in antenna chamber and need a rotation configuration to measure through the cross-track direction. Thus the ground-based experiment is not of such convenience and require stringent experiment environment. In the paper a configuration called image simulator is present. It could calibrate the receiver of the instrument conveniently and reliably. We describe the structure of the image simulator after analyzing the inversion algorithm. And then, briefly discuss one radiometric source simulation in the case of two dimensional synthetic aperture. Finally we give the G-matrix result of the x-band synthetic aperture radiometer.

Inversion and Calibration Equations

For one dimensional synthetic aperture microwave radiometer, two antennas of each pair received radiometric signals and the cross-correlation result can be written as

\[
<v_i v_j^*> = \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} v_T(\theta,t) v_T^*(\theta',t - \frac{d\sin\theta'}{c}) e^{-2\pi f \frac{d\sin\theta'}{c}} d\theta d\theta' \nabla = \int_{-\pi/2}^{\pi/2} v_T(\theta,t) v_T^*(\theta,t - \frac{d\sin\theta}{c}) e^{-2\pi f \frac{d\sin\theta}{c}} d\theta
\]
where $v_T(\theta)$ is the voltage seen at the antenna terminals, $\theta$ is the direction of incident radiation and $\omega$ is the observation frequency. $\langle, \rangle$ is the expectation operator. Noting that $v_T(\theta)$ and $v_T(\theta')$ are independent since they are generated by independent thermal sources of zero mean, the expectation collapse to a single integral. Moreover, $T(\theta)$ is proportional to $(v_T(\theta,t)v_T^*(\theta',t - d \sin \theta /c))$ and separate equation (1) into its real and imaginary components gives (where $d$ is the baseline distance)

$$V_I(d) = \int_{-\pi/2}^{\pi/2} T(\theta) \cos(2\pi fd \frac{\sin \theta}{c}) d\theta$$

$$V_Q(d) = \int_{-\pi/2}^{\pi/2} T(\theta) \sin(2\pi fd \frac{\sin \theta}{c}) d\theta$$

$V_I$, $V_Q$ are respectively the real and imaginary components of complex visibility functions. If it is thinned arrays of antenna contained many pairs of interferometer viz. $d = n \Delta u (n=0, 1, 2...N, N$ is corresponding to maximal antenna separation) , equation (2) becomes the Fourier harmonics of the FOVs.

But this analysis assumes identical antenna patterns and identical gain and phase parameters through each correlator channel. Actually, the instrument doesn’t meet such assumptions. Differences in correlator transfer functions cause two correlators measuring the same visibility function to have different outputs.

In the whole, considering all the effects of the instrument, substitute a Dirac delta function for $\theta$ of (3). The result is the impulse response of the system for a given baseline $n$ and angle $\theta$.

$$g(n, \theta_0) = c_{ij} \int_{-\pi/2}^{\pi/2} \delta(\theta - \theta_0) p_i(n, \theta) p_j(n, \theta) \exp(\frac{2\pi}{\lambda} n d \sin \theta) d\theta$$

where $p_i(\theta)$ and $p_j(\theta)$ are respectively antenna patterns of antenna i and j. $c_{ij} = |c_{ij}| e^{j \phi_{ij}}$ is complex gain of correlator between channel i and channel j. Phase errors between channels is critical in correlation radiometer visibility functions, the equation (2) becomes,

$$V_I(n) = \int_{-\pi/2}^{\pi/2} g_a(n, \theta) g_r(n, \theta) T(\theta) d\theta$$

$$V_Q(n) = \int_{-\pi/2}^{\pi/2} g_a(n, \theta) g_Q(n, \theta) T(\theta) d\theta$$

where $g_a(n, \theta)$ represents antenna pattern, $g_r(n, \theta)$ represents complex gain between channels, superscript I, Q represent odd and even lines G matrix. Measuring $g(n, \theta)$ in discrete angle $\theta$, then

$$V(n) = G_a G_r T$$

where $V$ is visibility function samples, $T$ is m-d column vector, $G_r = (g_{r1}, g_{r2}, ..., g_{rm})$, $G_a = \text{diag}(g_{a1}, g_{a2}, ..., g_{am})$. Variable m is determined by special sample and generally let $m > 2n + 1$, we take $m > 3n$.

The effect of antenna arrays is invariable with time and only depend on the structure of the antenna arrays and can be measured. So next we only consider the effect of the receiver. Rewriting equation (6) with lumping in-phase and quadrature-phase together yields

$$V = GT$$

where $V$ is a vector with p elements, $p = 2n + 1$ is results of all correlators, $T$ is vector with m elements, $G$ is $(p \times m)$ matrix.

Brightness temperature image reconstructed is as follows

$$\hat{T} = \tilde{G}^{-1} V$$

where $\tilde{G}^{-1} = G^* [\tilde{G} G^*]^{-1}$ is pseudo-inverse of G and T is the estimated brightness image.
1. System Design

Image simulator has the ability of simulating one point noise source in arbitrary direction in the FOV. Therefore the simulator can be used measuring the G matrix of the synthetic aperture radiometer.

While worked in calibration mode the system uses pre-computed simulation data to control eight RF signals which simulates equivalent antenna signals of the point source from different angles in the far field. Then we can get G matrix exclude the antenna effect by measuring the outputs of the radiometer. Considering the number of reconstruction is so many as 156, so the simulator is designed working under the control of digital interface of the synthetic aperture radiometer.

We take X band 8 channel synthetic aperture microwave radiometer for example. Figure 1 is the block diagram of the image simulator. The system is constructed with power divider network linked to a signal source and 8 IQ vector modulators. RFI signals from one source go through power divider to become 8 outputs Rfin1∼Rfin8, then be input into 8 digital 12bit IQ vector modulators and shift the phase of the signal to gain 8 outputs RFO1∼RFO8 which replace antenna signals to be input into the synthetic aperture radiometer. Power divider and digital IQ vector modulator are available. Therefore the main part of the design of the circuitry is to generate the control word of the 8 12bit IQ vector modulators and correspond with the digital interface of the synthetic aperture radiometer.

Digital IQ vector modulators can be calibrate to provide precision control on both amplitude and phase of the transmitted signal simultaneously. To obtain the highest degree of accuracy, the calibration should be performed in-situ. Calibration procedure of the device is performed using a vector network analyzer and a generated test program which employs an iterative algorithm to achieve the utmost in accuracy. Each IQ vector modulator has 24bit control word. Calibration program can provide amplitude accuracy of ±0.2 dB and phase accuracy of ±1.0 degrees.

Under the calibration mode, digital manage subsystem’s MCU first send a request signal to the simulator, then the simulator’ MCU read the control word (24bit) corresponding to one of eight antenna signal and send to the corresponding IQ vector modulator. Finally, the simulator send back a signal to the digital manage subsystem to inform the completed state. Digital manage subsystem collects outputs of the synthetic aperture radiometer and transmitted to the computer and save them until all point source are simulated.

2. Test Result

Figure 2 shows interference patterns for $d = n \Delta u (n = 0, 1, 2, 3)$ spacing and both I and Q channels (due to the length limit of the paper). The data have been normalized. The frequency of the oscillations results from the antenna separation. These visibility functions are the basis functions used to reconstruct brightness temperature scenes and are the rows of the G-matrix. The figure visualized shown the amplitude difference among each baseline and also shown the difference between the in-phase and the quadrature components. The reason is that every channel and every correlator has unique transmission function.

Simulating 156 point source in the far field and reconstructing the brightness temperature scenes can produce the spatial impulse response of synthetic aperture radiometer.
\[ T_\delta = \tilde{G}^{-1} G \] (8)

where the measured visibilities act as both the G-matrix and the visibilities that are to be inverted. The measured result closely resembles the identity matrix shown as Figure 3. The nadir spatial impulse response is the center row or column of the impulse response matrix. Figure 4 shows the nadir spatial impulse response of the synthetic aperture radiometer.

3. Two Dimensional G-matrix

For two dimensional synthetic aperture microwave radiometer, the visibility function and the brightness temperature are related as [1], [2]:

\[ V(u, v) = K \int_{\xi^2 + \eta^2 \leq 1} T(\xi, \eta) e^{-j2\pi(u\xi + v\eta)} d\xi d\eta \] (9)

\[ T(\xi, \eta) = \frac{T_B(\xi, \eta)}{\sqrt{1 - \xi^2 - \eta^2}} |F_n(\xi, \eta)|^2 \] (10)

where \((u, v)\) is the spacing between the two antennas in wavelengths; \(T_B(\xi, \eta)\) is the brightness temperature; \(T(\xi, \eta)\) will be called the modified brightness temperature; \(1/\sqrt{1 - \xi^2 - \eta^2}\) is the obliquity factor for a thermal source lying on a plane; \(F_n(\xi, \eta)\) is the normalized antenna voltage pattern; \(\xi = \sin \theta \cos \phi\), \(\eta = \sin \theta \sin \phi\), are the directing cosines. In the same way, image simulator can also simulate a point source in \(\xi - \eta\) plane. Assume there are 64 x 64 pixels in the \(\xi - \eta\) plane, G-matrix is \(G(u_n, v_n, \xi_m, \eta_m)\) where \(n\) is baseline number corresponding to spatial frequency sample; \(m = 64 \times 64 = 4096\) is pixel number in the FOV, i.e. it is equivalent
to 64 one dimensional G-matrixes, and there are 64 samples in each one dimensional direction. Hence, it is obvious that the computation is huge.

**Conclusion**

Calibration of the synthetic aperture radiometer is the most important issue after the instrument is developed. It reflects the true relation between the output (visibility) and the brightness temperature at the input. It also effects the image reconstruction algorithm. In this paper, a measurement set up called image simulator is present. This subsystem can expediently simulate one point noise source in arbitrary direction in the FOV and measure the G-matrix to describe the instrument exclude antenna arrays and using for image reconstruction. The simulator was tested with X band 8 channel synthetic aperture microwave radiometer, the experiment testify the theory in section two.

**REFERENCES**