Model for Differential Model Delay Distribution in Multimode Fibers

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Abstract

A band-limited fixed frequency linearly polarized optical signal excites several modes of propagation when it is launched into a multi-mode dimensioned optical fiber waveguide. This signal will therefore propagate over multiple paths along the transmission medium resulting in different propagation time for each mode. Thus replicas of the input pulse launched into the multimode fiber arrive at the output at different times, with the fundamental mode arriving first. A Differential Modal Delay (DMD) distribution is thus produced at the output of the fiber. The mode distribution, and consequently the modal delay distribution, are both a function of the physical attributes (geometry, distance, launching angle) of the optical waveguide. In optical communication systems this DMD distribution creates signal distortion that limits system designs (power, modulation, noise) and network performance (reach, rate, capacity). Accurate quantification of this DMD distribution is therefore essential to the prediction and improvement of performance.

Introduction

Multimode fibers guide light in multiple modes. For a given frequency, each of these modes propagates with a different velocity. As a result, each mode travels through the fiber with a different propagation time [3]. A pulse that is launched into the multimode fiber appears at the output multiple times, slightly delayed with respect to the fundamental mode and each other. The resulting pulse formed by overlap of the multiple images of the original pulse is distorted in shape and time. This effect is known as multimode distortion, and the differential delay that is responsible for this kind of distortion is known as differential modal delay (DMD).

Experiments with the existing models for DMD show unclear and conflicting results to capture and quantify this distribution. Two standard models are the Normal and WKB (Wentzel, Kramers, Brillouin) and the delay associated with each mode is characterized by:

\[ \tau_{\text{NORMAL}} = \frac{L}{c} \left( N_{\text{eff}} - \frac{\lambda_0}{k_0} \frac{dN_{\text{eff}}}{d\lambda_0} \right) \]
\[ \tau_{\text{WKB}} = \frac{L n_1}{2c} \left( \frac{n_1}{N_{\text{eff}}} - \frac{N_{\text{eff}}}{n_1} \right) \]

Where \( N_{\text{eff},i} = \frac{\beta_i}{k_0} \), \( k_0 = \frac{2\pi}{\lambda_0} \) (the free-space wave number), and \( \beta_i \) = the modal propagation constant for the \( i^{th} \) mode [9].

These models have produced differing delay distributions. Thus it is not clear which model would more accurately characterize DMD and its effects.

Model for DMD

A method to effectively model the modal delay is investigated in this paper. The methodology uses a simple ray tracing approach to determine all possible modes that can be supported by a multimode waveguide with defined physical parameters such as core refractive index, waveguide thickness, fiber length and operating wavelength [4]. This is a simple and straightforward approach that does not require the solution of Maxwell’s equations for a particular multimode fiber [1]. Each supported optical mode is modeled as a transverse field profile with unique transit time [2,3]. The waveguide-supported family of modes will thus completely define the delay distribution of the guided modes. The effect of variation of the above-mentioned physical parameters will also be considered in understanding and illustrating the modal delay spread and delay distribution at the output. This approach does not require solutions of the wave equations for the multimode transmission medium. It will characterize the transmission medium in terms of the output delay distribution. Moreover, it will generate a realistic delay distribution model in order to quantify the impact of modal distortion on optical network performance.
The plot in Figure 1 shows the modal distribution in a waveguide of given material and geometry [1]. Each intersection in the graph corresponds to a guided mode within the waveguide. The coordinates of the intersection give us the values of $kx_1$ and $\gamma_2$. $kx_1$ is the value of propagation constant in the $x$-direction and in a medium of refractive index $n_1$. $\gamma_2$ is the attenuation coefficient in a medium of refractive index $n_2$. Since $kx_1$ and $\gamma_2$ are multiplied by $d$, which is the thickness of the guide, this is a normalized plot for a symmetric slab waveguide of any width. Figure 1 shows four intersections producing two Symmetric and two Antisymmetric TE modes in the waveguide. Figure 2 shows the corresponding electric field distribution. Four distinct modes are excited. The $m$th mode has $m$ zeroes associated with it. Thus, these figures clearly show the excitation of more modes with the increase in the value of $R$ ($V$-number) in a symmetric dielectric slab waveguide. $R$ is given by $ko_d^2(n_1^2 - n_2^2)$, and $k0 = \frac{2\pi}{\lambda} = \omega\sqrt{\mu_0\varepsilon_0}$ = free-space propagation constant, which satisfies the expression: $(kx_1d)^2 + (\gamma_2d)^2 = ko_d^2(n_1^2 - n_2^2)$.

It is clear from the example and plots shown that the graphical solutions given by the crossing points in the curves is a simple and easy way to determine the modal solutions for different values of $R$. Each fixed value of $R$ represents a fixed waveguide geometry, material and operating frequency. A variation in any of these parameters (waveguide geometry, material and operating frequency) will give rise to a different value of $R$, hence a different set of modal solutions. For a given value of $R$, the discrete series of solutions obtained: $kx_1, kx_1, kx_1, \ldots, kx_1_n$ will have a corresponding discrete series of incident angles: $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$, related to each value of $kx_1$, where $kx_1 = k_1 \cos \theta_1$. $\theta_1$ is the angle made by the incident ray with the normal at the core-cladding interface of the waveguide. This means that each excited mode within the guide is a function of a pre-determined launching angle. For each launching angle that is associated with an excited mode within the guide, ray trajectories can be constructed by following the zigzag paths of the totally reflected ray at each reflection [1,4] (Snyder and Love: Optical Waveguide Theory). Since such rays are entirely confined within the core, these are referred to as bound rays. It is assumed that with each reflection of a bound ray, the power also gets totally internally reflected within the core [4]. Hence, bound rays can propagate indefinitely without any loss of power.

The ray transit time, $t$ is defined as the time taken by a ray to travel a distance $z$ along the waveguide axis following repeated series of zigzag paths caused by the well-known theory of total internal reflection [4]. The ray transit time for a given mode (ray-path) can be estimated by the following expression, which is a function of the guide geometry, material and operating wavelength.

$$t = \frac{zn_1}{c\sqrt{1 - \frac{\lambda^2 n^2}{4zd}}} \tag{2}$$

The ray transit time will in turn be used as an important parameter to estimate the DMD profile for the guide.

The general expression for the relative differential delay of a mode (relative to the delay/transit time of the fundamental mode) is given by:

$$\Delta t_i = t_i - t_0 \tag{3}$$
\[ \Delta t_i = \frac{2n_1}{c \sqrt{1 - \frac{\lambda_1^2 m_i^2}{4d^2}}} - \frac{2n_1}{c} \quad \text{for} \quad \frac{\lambda_1^2 m_i^2}{4d^2} < 1 \] (4)

The general expression for the consecutive differential delay of a mode is given by:

\[ t_{i+1} - t_i = \frac{2n_1}{c \sqrt{1 - \frac{\lambda_2^2 m_i^2}{4d^2}}} - \frac{2n_1}{c \sqrt{1 - \frac{\lambda_2^2 m_i^2}{4d^2}}} \] (5)

Figures 3, 4, 5 and 6 show plots of delay distribution obtained from the DMD expression discussed above. The effect of variation in the length of the waveguide (z) is shown. The intensity distribution of the modes has not been taken into consideration, and in all the cases that follow, normalized amplitudes have been assumed.

The DMD plots shown above reveal modal distribution that is nonlinearly spaced in time. The delay distributions both shift and expand with distance. This means that as the length of the channel increases, the distortion envelope causes a pulse to both expand and distort due to the nonlinear overlap of multiple replicas of the pulse at the output.

With the help of such DMD models, one can therefore have knowledge of the pulse distortion at the other end of the fiber. This kind of analysis is of extreme importance to design an appropriate receiver structure for optimization of signaling in a channel that suffers from DMD.

**Comparison of Various Approaches**

The WKB and the Normal method are two different standard models that are used to estimate the modal distribution in multimode fibers suffering from the effect of differential modal delay. The delay associated with each model is characterized by:

\[ \tau_{\text{NORMAL}} = \frac{L}{c} \left( N_{eff} - \lambda_0 \frac{dN_{eff}}{d\lambda_0} \right) \quad \tau_{\text{WKB}} = \frac{Ln_1}{2c} \left( \frac{n_1}{N_{eff}} \right) \frac{N_{eff}}{n_1} \] (6)
Where, \( N_{\text{eff},i} = \frac{\beta_i}{k_0}, \quad k_0 = \frac{2\pi}{\lambda_0} \) (the free-space wave number), and \( \beta_i \) = the modal propagation constant for the \( i^{th} \) mode.

These are well-known methods of channel modeling, but have produced differing delay distributions [9]. Thus it is not clear which model would more accurately characterize DMD and its effects. The analysis produced by the Normal approach shows that the intermodal delay separation is largest for the first two modes and shrinks for higher order modes that have been excited for the particular wavelength. An interesting observation that can be made from the above plots is that the delay distribution produced by the DMD model is similar to the WKB approach. In both cases, for a waveguide of fixed geometry and a single given wavelength, the distribution envelope shifts in time with increasing waveguide length. In both cases, the last two modes experience the largest differential delay. However, study shows that when different single wavelengths are launched into a waveguide of fixed geometry and fixed length, the proposed DMD approach and the WKB approach produce distribution envelopes that do not agree with one another. In the WKB analysis, the distribution envelope does not change for various wavelengths, whereas in the new analysis the distribution envelope is different for different wavelengths as illustrated in the corresponding plots.

### Conclusion

The distribution envelope that is closest to the differential modal delay distribution is used to predict pulse distortion at the end of a channel of known material and geometry. The knowledge of the modal delay distribution in a channel that is subject to DMD is fundamental to the optimum design and application of adaptive receivers, delay equalizers and error correction in such channels. This paper provides a new and different approach to the characterization of DMD, which is dominant impairment to high-speed communications in multimode fibers over short-reach optical networks. The WKB and the Normal methods propose similar analysis of the channel. Initial comparison of results suggests that the approach of this paper is better-suited to the characterization of differential modal delay and subsequent estimation and optimization of link performance.

### REFERENCES

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