Backscattering from Multi-scale and Exponentially Correlated Surfaces

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Abstract

Most natural surfaces have been reported to have an exponential-like correlation function and generally contain more than one scale of roughness. In this paper we want to show that a multiscale Gaussian-distributed surface with a differentiable correlation function also possesses an exponential-like correlation function except near the origin. As a result, angular backscattering from such a surface in the low frequency region behaves the same as if it is from an exponentially correlated surface. In the high frequency region, scattering from the two differently correlated surfaces should be quite different as expected. Furthermore, we show that the differentiable correlation function for a multi-scale, Gaussian-distributed surface can be used to interpret backscattering from a known randomly rough surface that appears to possess an exponential correlation over a wide range of frequencies. These findings indicate that for natural surfaces the exponential correlation function may be a valid approximation to the real correlation in the low frequency region, although it is not the correct function in the high frequency region. It also offers an explanation as to why many natural surfaces appear to be exponentially correlated.

1. Introduction

In the study of surface scattering over the last forty years it has been found that most natural surfaces possess an exponential-like correlation function [Hayre and Moore, 1961; Oh et al., 1992]. In practice, best fit to data is often realized with an exponential correlation function. However, an exponential correlation function is not differentiable at the origin and hence cannot lead to a meaningful scattering model in the high frequency region. Furthermore, the use of an exponential correlation does not allow the backscattering coefficient to approach the geometric optics solution in the high frequency limit. Thus, an exponential correlation function cannot be the correct function for any surface scattering model in the high frequency region. One possible answer to this dilemma is that the surface is actually a multi-scale surface and each surface roughness scale possesses a differentiable correlation function. It is the combination of these scales that makes the overall correlation function appear exponential-like. The purpose of this paper is to demonstrate this possibility using a physical surface with measured surface parameters along with a set of radar measurements over a frequency range from 1.5 GHz to 9.5 GHz.

To provide the stated demonstration we select in Section 2 a set of backscattering data from a surface with a measured root mean squared height of 0.4 cm and a correlation length of 8.4 cm. This set of three-frequency measurements, 1.5, 4.75 and 9.5 GHz, was taken by Oh et al. [1992] and can be interpreted with the IEM surface scattering model [Fung, 2004] using the exponential correlation, \( (0.4)^2 \exp[-|r|/8.4] \). In Section 3, we show that by replacing the exponential correlation with another three-scale correlation function with an adjusted curvature, we can produce very similar backscattering angular curves at all three frequencies. This last result demonstrates that the true correlation function can be an exponential-like but differentiable correlation function. This demonstration also shows that a multi-scale surface tends to possess an exponential-like correlation, but the true correlation function may not be exponential. Conclusions are given in Section 4.

2. An Exponential-like, Correlated Surface

Many radar measurements have been taken by Oh et al. [1992] along with surface profiles from which the surface root mean squared (rms) height and its correlation length are computed. Due to the finite length of the measured profiles, the complete shape of the correlation function cannot be determined, but the general appearance of the function has an exponential look as given in Figure 2 of Oh et al. [1992]. For ease of reference we repeat Oh et al.’s reported correlation function in Fig.1, where the corresponding exponential and Gaussian functions with the same correlation length are also plotted for comparison. It is seen that over a distance close to...
Figure 1: Comparisons of the correlation function based on measured profile and an exponential and Gaussian function according to Oh et al. [1992].

twice the correlation length, the reported correlation function can be approximated by the exponential function.

The general form of the vertically, \( \sigma^0_{vv} \), and horizontally, \( \sigma^0_{hh} \), polarized backscattering coefficients based on the integral equation method [3] is

\[
\sigma^0_{pp} = \frac{k^2}{4\pi} \exp[-2k^2 \sigma^2 \cos^2 \theta] \sum_{n=2}^{\infty} |I^n_{pp}|^2 \frac{w^{(n)}(2k \sin \theta, 0)}{n!}
\]

(1)

where \( p = h \) or \( v \).

\[
I^n_{pp} = (2k\sigma \cos \theta)^n f_{pp} \exp[-k^2 \sigma^2 \cos^2 \theta] + (k\sigma \cos \theta)^n F_{pp}
\]

\[
f_{vv} = (2R_v)/(\cos \theta), \quad f_{hh} = (-2R_h)/(\cos \theta)
\]

\[
T_p = 1 + R_p, \quad T_{pm} = 1 - R_p, \quad sq = \sqrt{\mu_r \varepsilon_r - \sin^2 \theta}
\]

\[
F_{vv} = \left( \frac{\sin^2 \theta}{\cos \theta} \right) \frac{sq}{\varepsilon_r} T^2_v - 2 \sin^2 \theta \left( \frac{1}{\cos \theta} + \frac{1}{sq} \right) T_v T_{vm} + \left( \frac{\sin^2 \theta}{\cos \theta} + \frac{\varepsilon_r (1 + \sin^2 \theta)}{sq} \right) T^2_{vm}
\]

\[
F_{hh} = -\left[ \left( \frac{\sin^2 \theta}{\cos \theta} \right) \frac{sq}{\mu_r} T^2_h - 2 \sin^2 \theta \left( \frac{1}{\cos \theta} + \frac{1}{sq} \right) T_h T_{hm} + \left( \frac{\sin^2 \theta}{\cos \theta} + \frac{\mu_r (1 + \sin^2 \theta)}{sq} \right) T^2_{hm} \right]
\]

In the above, \( k \) is the wave number; \( \sigma \) is the rms surface height; \( R_p \) is the \( p \)-polarized Fresnel reflection coefficient; and the quantity \( w^{(n)} \) is the surface spectrum corresponding to the two-dimensional Fourier transform of the surface correlation coefficient \( \rho(x,y) \) raised to its \( n^{th} \) power, \( \rho^{(n)} \), defined as follows in polar form:

\[
w^{(n)}(\kappa, \varphi) = \int_0^{2\pi} \int_0^\infty \rho^n(r, \phi) e^{i\kappa r \cos(\varphi - \phi)} r dr d\phi
\]

(2)

If the surface roughness is independent of the view direction, the correlation coefficient is isotropic depending only on \( r \). In this case (2) reduces to

\[
w^{(n)}(\kappa) = 2\pi \int_0^\infty \rho^n(r) J_0(\kappa r) r dr
\]

where \( J_0(\kappa r) \) is the zeroth order Bessel function. The first term in \( I^n_{pp} \) is the Kirchhoff term and the second term is the complementary term.

A comparison of the IEM surface scattering model given by (1) with Oh et al.’s measurements from a wet surface with an rms height, \( \sigma \), of 0.4 cm and a correlation length, \( L \), of 8.4 cm at 1.5, 4.75 and 9.5 GHz using the exponential correlation is shown in Figures 2a through 4a. By changing the correlation to a three-scale Gaussian as

\[
G_{sq}(r) = \sigma^2 \left\{ \frac{\sigma_1^2}{\sigma^2} \exp[-(r/L_1)^2] + \frac{\sigma_2^2}{\sigma^2} \exp[-(r/L_2)^2] + \frac{\sigma_3^2}{\sigma^2} \exp[-(r/L_3)^2] \right\}
\]

(3)

where \( \sigma_1 = 0.79\sigma \), \( \sigma_2 = 0.43\sigma \), \( \sigma_3 = \sqrt{1 - \sigma_1^2 - \sigma_2^2} \), \( L_1 = 1.5L \), \( L_2 = 0.45L \), and \( L_3 = 0.14L \), leads to comparisons shown in Figures 2b, 3b and 4b.
It is seen that the use of the normalized correlation function, \( \exp[-|r|/8.4] \), leads to excellent agreements between model predictions and data at all three frequencies and over a wide range of incident angles. As indicated in Fung [2004], the IEM model agrees with data over a wider range in the incident angle than the empirical model provided by Oh et al. [1992]. Oh et al. [1992] indicated that the data at 10 degrees and 1.5 GHz is very high due to contributions from coherent scattering. Thus, the disagreement there is to be expected, because all scattering models are designed to account for incoherent scattering. In the next section we shall show that a differentiable correlation function from a surface with three scales of roughness can generate backscattering similar to \( \exp[-|r|/8.4] \).

Figure 2: Comparisons between backscattering data by Oh et al. at 1.5 GHz with the IEM model using the correlation function defined by (a) exponential and (b) \( G_{gg} \).

Figure 3: Comparisons between backscattering data by Oh et al. at 4.75 GHz with the IEM model using the correlation function defined by (a) exponential and (b) \( G_{gg} \).

Figure 4: Comparisons between backscattering data by Oh et al. at 9.5 GHz with the IEM model using the correlation function defined by (a) exponential and (b) \( G_{gg} \).

3. A Three-scale \( x \)-power Correlated Surface

A differentiable, \( x \)-power correlation function for three-scale randomly rough surfaces is defined to be,

\[
G_{pp}(r) = \sigma^2 \left\{ \frac{\sigma^2}{\sigma^2} \exp[-(r/L_1)^2] + \frac{\sigma^2}{\sigma^2} [1 + (r/L_2)^2]^{-x} + \frac{\sigma^2}{\sigma^2} [1 + (r/L_3)^2]^{-x} \right\}
\]

where \( x > 1.0 \). All other model parameters remain unchanged as in the previous section. Again, \( \sigma = 0.4 \) cm and \( L = 8.4 \) cm. A comparison between this correlation function with \( x = 1.2 \) and the \( G_{gg} \) is given in Fig. 5.
In Fig. 5, we see visually that $G_{pp}$ is almost identical to $G_{gg}$. The difference in curvature between the two functions cannot be seen by looking at the curves. We do know that $G_{pp}$ has a larger curvature and we expect it will produce much better fit to the data at higher frequencies. The reason why $G_{gg}$ cannot work is because an increase in curvature for $G_{gg}$ can only come at the expense of forcing the shape of the curve to depart significantly from the exponential function.

Figure 5: A comparison between the three-scale Gaussian correlation function, $G_{gg}$, and the three-scale $x$-power correlation function, $G_{pp}$, with $x = 1.2$.

In Fig. 6 we show comparisons with data at 1.5 GHz. The comparison based on the three-scale Gaussian correlation is given in part (a) and a similar comparison using $G_{pp}$ is in part (b). The backscattering curves are very similar, even though the correlation functions are different, and they all fit the data. The use of $G_{pp}$ gives a slightly better overall fit.

Figure 6: Comparisons between the backscattering data by Oh et al. at 1.5 GHz with the IEM model using (a) the $G_{gg}$ and (b) the $G_{pp}$ correlation functions.

To show that the model predictions using the $x$-power correlation function $G_{pp}$ can match data fairly well at the intermediate frequency of 4.75 GHz, we show in Fig. 7 similar comparisons as we did in Fig. 3. To see the improvement using this correlation function over the use of $G_{gg}$ we repeat the comparison using $G_{gg}$ in part (a) of Fig. 7. Clearly, part (a) in Fig. 7 shows a poorer fit to data for $vv$ polarization as compared to part (b) where the correlation function $G_{pp}$ is used.

To show that the $G_{pp}$ correlation continues to work at high frequencies we show in Fig. 8 similar comparisons as in Fig. 7 but at 9.5 GHz. In part (a) of Fig. 8 we show the comparisons using the $G_{gg}$ correlation and in part (b) the $G_{pp}$ correlation. We see that despite the two functions appear very close to each other as seen in Fig. 5, there is a huge difference in backscattering because these two functions represent different surface curvatures. An excellent agreement between model and data is realized only in part (b) of Fig. 8 with the $G_{pp}$ correlation.

The gradual shifting of backscattering from a dependence on the shape of the surface correlation function to a dependence on both the surface curvature and the shape of the function is observed. This is the reason why the use of $G_{gg}$ cannot lead to satisfactory backscattering results. The fact that the two functions $G_{pp}$ and $G_{gg}$ are so close as shown in Fig. 5 suggests that it is difficult to determine the curvature of the surface through measured surface profiles.

4. Conclusion

The comparisons between model and measurements shown in the previous section have shown the following:
1. A possible reason why most natural surfaces seem to be exponentially correlated is because they contain more than one scale of roughness.

2. Over a wide range of frequencies a multi-scale Gaussian correlated surface may appear to be exponentially correlated, when in fact it is not an exponential function and may have a finite curvature.

3. By allowing the large scale to be Gaussian correlated and the smaller scales to follow a power-law, a three-scale correlation function can approximate the exponential function quite well in terms of backscattering behavior over a wide frequency range.

4. The exponential correlation function could be viewed as a practical approximation to the true correlation function in the low and intermediate frequency regions.

5. The curvature of the surface is an important consideration in scattering especially in the intermediate and high frequency region.

REFERENCES


