Coupling of a Gaussian Beam into a Planar Slab Waveguide using the Mode Matching Method

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Abstract

The coupling of an optical beam into a dielectric multimode waveguide is investigated as one of the aspects in the modeling of optical interconnects. In order to obtain results as accurate as possible, the mode matching method is used with a high number of guided and evanescent modes on both sides of the face surface of a simple guiding structure – a planar slab waveguide. The power coupled into the guided modes of the waveguide has been calculated under variation of the parameters of an incident Gaussian beam, i.e., the beam width, the angle of incidence, and the lateral displacement. Also the properties of the waveguides have been varied. Some results are discussed.

1. Introduction

For modeling the transmission of optical waves through optical multimodal interconnects it is important to know which portion of the power irradiated on the front surface of a waveguide is coupled into its guided modes. There are several alternatives of modeling the coupling process. In order to get a reference for assessing simple methods the chosen method in this paper is the mode matching method taking a large number of modes into account. The mode matching method can be rather extensive but promises a very high accuracy.

The considered geometry is a planar slab waveguide with an asymmetric step index profile bordering the free space in a plane interface. Perfectly conducting walls are placed sufficiently far away from the interesting region to obtain discrete solutions for all kinds of modes. The electromagnetic field is independent of one transverse coordinate and is expressed in terms of transverse magnetic (TM) or transverse electric (TE) modes. The exciting field is a Gaussian beam which only depends on the longitudinal and one transverse coordinate and is either transverse electric or magnetic. It is expanded into the modes of a hollow waveguide formed by the conducting walls by means of a simple discrete Fourier transform. As the calculations for TM and TE modes are similar, only the TM case is presented here.

2. Modes of the Waveguides

Figure 1 shows the considered geometry in detail. The core of the slab waveguide with thickness \( d \) is placed between two perfectly conducting walls by a spacing of \( a \) on each side. The profile of the permittivities of the slab waveguide is asymmetric with \( \varepsilon_1 \) in the core, \( \varepsilon_2 \) in the substrate and \( \varepsilon_3 \) in the superstrate. The region in front of the slab waveguide \((z<0)\) can be considered as a simple planar hollow waveguide formed by the conducting walls.

The complex amplitudes of the time-harmonic electromagnetic fields must satisfy the Helmholtz equation

\[
\Delta(H, E) + \beta_i^2(H, E) = 0, \quad \text{with} \quad \beta_i^2 = \omega^2 \mu_i \varepsilon_i \quad \text{and} \quad i = 0, 1, 2, 3.
\]

Solutions of the Helmholtz equation for the hollow waveguide \((z<0)\) are the well-known modes

\[
H = e_x C \cos(\beta_z r (x - \frac{d}{2} - a)) e^{\text{exp} z}, \quad E = e_x \left( \frac{\beta_z}{\omega \varepsilon_0} \right) H + e_z \frac{1}{j \omega \varepsilon_0} \frac{\partial}{\partial x} H,
\]

with \( \beta_z^2 = \beta_0^2 - \beta_i^2 \) and \( \beta_i = \frac{n \pi}{d + 2a}, \quad n = 0, 1, 2, 3 \mathbb{K} \)
If $\beta_z$ is real, the upper sign in the argument of the exponential function denotes a propagation in the positive $z$-direction. $n$ is the mode index. As long as $\beta_z^2 - \beta_x^2 > 0$ is valid, the solutions are guided modes. To get the complete set of solutions, also the evanescent modes with $\beta_0 < \beta_z$ have to be regarded.

The modes in the slab waveguide ($z > 0$) are given by

$$H = e_z X(x) e^{-\beta_z z}, \quad \text{with}$$

$$X(x) = \begin{cases} 
C_3 \cosh(\beta_{z,3}(x - d/2 - a)) & \text{if} \quad d/2 \leq x < d/2 + a \\
C_1 \cos(\beta_{x,1}(x - d/2)) + D_1 \sin(\beta_{z,1}(x - d/2)) & \text{if} \quad -d/2 \leq x < d/2 \\
C_2 \cosh(\beta_{z,2}(x + d/2 + a)) & \text{if} \quad -d/2 - a \leq x < -d/2
\end{cases}$$

(3)

and $\beta_{z,3} = \beta_z^2 - \beta_x^2$, $\beta_{z,3} = \beta_z^2 - \beta_x^2$, $\beta_{z,3} = \beta_z^2 - \beta_x^2$.

Because of the boundary conditions in $|x|=d/2$ and $|x| = d/2+a$, the constants $C_1, C_2$ and $C_3$ depend on $D_1$ according to

$$C_1 = -\frac{\varepsilon_z}{\varepsilon_1} \frac{\beta_{z,1}}{\beta_{z,3}} \tanh(\beta_{z,3} a), \quad C_2 = -\frac{\varepsilon_z}{\varepsilon_1} \frac{\beta_{z,1}}{\beta_{z,3}} \sinh(\beta_{z,3} a),$$

$$C_3 = -\frac{D_1}{\cosh(\beta_{z,2} a)} \left( \frac{\varepsilon_z}{\varepsilon_1} \frac{\beta_{z,1}}{\beta_{z,3}} \cos(\beta_{z,2} d) + \sin(\beta_{z,2} d) \right).$$

(4)

The constant $D_1$ remains undetermined and is the amplitude of the mode. The phase constants must satisfy the eigenvalue equation

$$\frac{\varepsilon_1}{\varepsilon_2} \beta_{z,1} \tanh(\beta_{x,2} a)/\varepsilon_2 = \frac{\varepsilon_3}{\varepsilon_1} \beta_{z,1} \tan(\beta_{z,2} d) - \frac{\varepsilon_3}{\varepsilon_1} \beta_{x,3} \tan(\beta_{z,2} a)$$

(5)

The solutions of (5) can best be found by expressing $\beta_{z,2}$ and $\beta_{x,3}$ in terms of $\beta_{x,1}$. Solutions in the interval $[0,\sqrt{\beta_x^2 - \beta_z^2}]$ are guided modes. For $\beta_{x,1}$ in $[\sqrt{\beta_z^2 - \beta_x^2}, \sqrt{\beta_x^2 - \beta_z^2}]$ the phase constant $\beta_{z,2}$ becomes imaginary. The accompanying modes are called radiation modes of type I. Radiation modes of type II belong to solutions $\beta_{x,1}$ in the interval $[\sqrt{\beta_z^2 - \beta_x^2}, \beta_x]$ ($\beta_{z,2}$ becomes imaginary). Also the evanescent modes with $\beta_1 < \beta_z$ are needed. An algorithm for finding all solutions of (5) can be engineered using the known singularities of the tangent function.

3. Mode Matching

The mode matching method employs the boundary conditions of the transverse electric and magnetic fields in $z = 0$ to determine the amplitudes of the reflected and transmitted modes. The boundary conditions are given by

$$\sum_n \left( C_n^{(i)} E_{0n}^{(i)} + C_n^{(r)} E_{0n}^{(r)} \right) = \sum_n C_n^{(i)} E_{0n}^{(i)},$$

$$\sum_n \left( C_n^{(i)} H_{0n}^{(i)} + C_n^{(r)} H_{0n}^{(r)} \right) = \sum_n C_n^{(i)} H_{0n}^{(i)}.$$ 

(6)

(7)

The variables $E$ and $H$ are the mode functions from Sec. 2, but normalized and evaluated in $z = 0$, so that

$$\frac{1}{2} \left( \int_{z=0}^{r} E_{0n}^{(i)} \times H_{0n}^{(i)} e_z \, da = \delta_{mn}, \quad l \in \{i, r, t\} \right) \quad \text{with} \quad \delta_{mn} = \begin{cases} 
\text{mj} & \text{if} \quad n = m, \text{evan. modes} \\
\pm 1 & \text{if} \quad n = m, \text{other modes} \\
0 & \text{else}
\end{cases}$$

(8)

is valid. Therefore the constants $C_n^{(i)}$ are normalized amplitudes. The upper index $i, t, r$ denotes incident, transmitted, or reflected modes, respectively, while the lower index $n$ is the mode number, and $x, y$ denote the components of the electric and magnetic fields. The upper signs in the definition of $\delta_{mn}$ denotes a propagation in the positive $z$-direction.

Scalar multiplication of (6) with $H_{0n}^{(i)}/2$ and (7) with $E_{0n}^{(i)}/2$ and integration over the interface $z = 0$ of both equations results with consideration of (8) in.
\[ \sum_n \left( C_n^{(i)} - C_n^{(r)} \right) \mathbf{I}_{n,k} = C_k^{(i)} \left\{ \begin{array}{c} 1 \\ j \end{array} \right\} \quad \text{and} \quad \left( C_k^{(i)} + C_k^{(r)} \right) \left\{ \begin{array}{c} 1 \\ -j \end{array} \right\} = \sum_n C_n^{(r)} \mathbf{I}_{n,k}^{(r)} \quad \text{with} \quad I_{n,k} = \frac{1}{2} \int \mathbf{E}^{(i)}_{m} \mathbf{H}^{(r)*}_{m} \, da. \quad (9) \]

The equations in (9) represent a linear system of equations with a solution vector consisting of the undetermined amplitudes \( C_n^{(i)} \) and \( C_n^{(r)} \). A similar equation system could be derived by multiplication of (6) with \( \mathbf{H}^{(r)*} / 2 \) and (7) with \( \mathbf{E}^{(i)}_{m} / 2 \). For exact calculations the number of modes in (9) must tend to infinity. Of course only a finite number of modes can be taken into account in a numerical calculation. Irrespective of the number of modes used to represent the fields, the mode matching equations in (9) preserve the power\(^2\) across the discontinuity in \( z = 0 \).

4. Gaussian Beam

The fields of the transverse magnetic Gaussian beam are described by\(^3\)

\[ \mathbf{H} = -\frac{j \beta}{\mu} \mathbf{e}_r U(x, z) e^{-\beta z}, \quad \mathbf{E} = -j \omega \left( \mathbf{e}_x + \frac{\mathbf{e}_r}{z} \right) U(x, z) e^{-\beta z} \]

with \( U(x, z) = A_0 \sqrt{\frac{2}{\pi w(z)^2}} e^{-\frac{x^2}{w(z)^2}} e^{\left( \frac{\beta z}{2(\nu z_0^2 + 1)} \right) z^2 \tan^{-1} \left( \frac{z}{z_0} \right)} \) and \( w^2(z) = \frac{2 z_0}{\beta} \left( 1 + \frac{z^2}{z_0^2} \right) \).

The variable \( w(z) \) represents the beam radius and depends on the Rayleigh range \( z_0 \) which is a measure for the angle of divergence. The quantity \( 2z_0 \) is called confocal parameter.

The fields of the Gaussian beam can be expanded into modes of the hollow waveguide (2) by means of a fast Fourier transform. In general, a trigonometric interpolation polynomial defined in \([x_0, x_0 + T]\) is given by

\[ I(x) = d_0 + d_k \cos \left( K 2\pi \frac{x - x_0}{T} \right) + \sum_{k=1}^{K} \left( d_k - d_{N-k} \right) \cos \left( K 2\pi \frac{x - x_0}{T} \right) + j \left( d_k - d_{N-k} \right) \sin \left( K 2\pi \frac{x - x_0}{T} \right) \]

with \( N = 2K \) sampling points. May \( g(x) \) be the function to be interpolated, then the Fourier coefficients \( d_k \) are defined by

\[ d_k = \frac{1}{N} \sum_{n=0}^{N-1} g_n e^{-jk2\pi x/N} \quad \text{with} \quad k = 0, K, N - 1 \quad \text{and} \quad g_i = g(i \frac{T}{N} + x_0). \quad (13) \]

When the function for the magnetic field in (11) is interpolated by a polynomial (12) with vanishing sine functions, it is possible to determine the amplitudes of the modes of the hollow waveguide by comparing the coefficients. Thus, the function for the magnetic field must be extended suitably periodically for \(|x| > a+d/2\) before the interpolation.

5. Results

Various calculations have been made using the described techniques with different angles of beam incidence and lateral displacement of the beam axis. To obtain an angle of divergence of \( 14^\circ \) the confocal parameter of the Gaussian beam has to be \( 2z_0 = 8.7 \mu m \). The wavelength has been set to a typical VCSEL wavelength of 850 nm. In order to realize different beam diameters \( b = 2w(z) \) in the interface \( z = 0 \) the source of the beam has been displaced in the negative \( z \)-direction. A core thickness of \( d = 100 \mu m \) and refractive indices of 1.56 in the core, 1.54 in the substrate, and 1.53 in the superstrate lead to 58 guided modes in the slab waveguide. The distance \( a \) and the number of modes needed for a good approximation of the coupling process have to be determined.

Calculations have shown that with a maximum angle of incidence of \( \theta = 20^\circ \) and a maximum lateral shift of the beam axis from the center of the waveguide core of \( h = 125 \mu m \) a distance \( a = 300 \mu m \) is sufficient. In this case a number of 1500 modes must be taken into account in both waveguides. For the definition of \( \theta \) and \( h \) see Figure 1.

Figure 2 presents exemplary results where either \( \theta \) (left figure) or \( h \) (right figure) have been varied. The results can be interpreted by means of a simple ray optical model where the Gaussian beam is approximated by a bundle of differently weighted and oriented rays. The numerical aperture of the
slab waveguide is given by $A_n = \sqrt{n_1^2 - n_2^2}$. Rays that hit the core interface with an angle less then $\theta_{\text{max}} = \arcsin(A_n)$ are guided within the core of the slab waveguide. If the beam width is less than the core thickness and the lateral shift is small, all rays hit the core. Furthermore, if the angle of incidence $\theta$ is small, most rays hit the interface with angles less then $\theta_{\text{max}}$ and the maximum of 91.3% of the emitted power is coupled into the guided modes. In case of large beam widths many rays don’t hit the core interface and the maximum power decreases. Small changes of $\theta$ don’t take effect because the amount of elementary rays that hit the interface with angles less then $\theta_{\text{max}}$ doesn’t decrease rapidly.

Figure 3 shows results where $\theta$ has been varied for different displacements $h$. In the right figure the refractive indices of the substrate and superstrate have been decreased. Ray optical interpretations are also possible.

Calculations with transverse electric fields yield to similar results for the given Parameters. A maximum relative deviation of 1% was observed for the results shown above.

6. Conclusion

Due to the efficiency of recent computer systems the mode matching method is a powerful method for investigating coupling processes at waveguide discontinuities. The approach described in this paper can easily be adapted to related problems as long as the modes of the participating waveguides form a discrete spectrum. With a large number of modes taken into account the mode matching method promises a very high accuracy. Thus, it may be used as a reference for assessing simpler methods.

REFERENCES