Light Scattering by Erythrocyte: Discrete Sources Method Model

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Abstract.
In this paper the Discrete Sources Method (DSM) is applied to model polarized light scattering by the human erythrocyte. We present several possibilities to model erythrocyte shape. Comparative results associated with light scattering by different erythrocyte shapes: disk-sphere and oblate spheroid are presented. The comparison with the results relating to the rigorous biconcave erythrocyte shape is also given.

Introduction
In recent years interest associated with polarized light scattering by different biological objects increases. Because biological structures are complex, it is generally difficult to study single scatterers. However there is an important exception, the red blood cell. First the erythrocyte is important for tissue optics because it contains one of the strongest absorbers of visual light in the human body, hemoglobin. Second, erythrocytes can easily be isolated and studied experimentally. Third, the erythrocyte does not show internal structure, providing an opportunity to apply theoretical models of scattering to these cells. Recently different methods were applied to model light scattering from a single erythrocyte: Wentzel-Kramer-Brillouin approximation, Mie theory, Fraunhofer and anomalous diffracton, Rayleigh and Rayleigh-Gans-Debye approximation, Fredholm Integral Equations, T-matrix and others. To model erythrocyte one usually uses the model of an oblate spheroid with aspect ratio 1:4, but the real erythrocytes have a biconcave discoid shape that complicates modeling and interpretation of light scattering. Especially there is a problem with T-matrix method to model particles with concavities. Some experimental studies were performed using a Scanning Flow Cytometer (SFC) that allows comparing the result of theoretical modeling with the experimental results [1].

As in frame of DSM [2] any axial symmetric object can be treated, we present a common model of oblate spheroid and disk-sphere. At the same time we modeled the real biconcave erythrocyte shape to compare with common models.

Theory
Let us start with the mathematical statement of the scattering problem. We will consider scattering in an isotropic homogeneous medium in \( \mathbb{R}^3 \) of an electromagnetic wave by a local homogeneous penetrable obstacle \( D_i \) with the smooth boundary \( \partial D \). We assume the time dependence to be \( \exp(j\omega t) \). Scattering is described by the electromagnetic fields \( \{E_{e,i}, H_{e,i}\} \) satisfying Maxwell equations:

\[
\nabla \times H_{e,i} = jk \varepsilon_{e,i} E_{e,i}, \quad \nabla \times E_{e,i} = - jk \mu_{e,i} H_{e,i}, \quad \text{in } D_{e,i}, \quad D_e := \mathbb{R}^3 / D_i,
\]

the transmission condition enforced on the particle surface:

\[
\nabla \times H_{e,i} \cdot n = 0, \quad \nabla \times E_{e,i} \cdot n = 0, \quad \text{on } \partial D_i.
\]
\[ \mathbf{n} \times (\mathbf{E}_i(p) - \mathbf{E}_e(p)) = \mathbf{n} \times \mathbf{E}_0, \]
\[ \mathbf{n} \times (\mathbf{H}_i(p) - \mathbf{H}_e(p)) = \mathbf{n} \times \mathbf{H}_0, \quad p \in \partial D, \]

and Silver-Muller radiation condition at infinity [3].

Here \( \{ \mathbf{E}_0, \mathbf{H}_0 \} \) is an exciting field, \( \mathbf{n} \) is the unit outward normal to \( \partial D \), index \( e \) belongs to the external domain \( D_e \), and \( i \) to domain inside the particle \( D_i \), \( \varepsilon_{e,i} \) is the permittivity, \( \mu_{e,i} \) - permeability, \( \text{Im} \varepsilon_e, \mu_e = 0 \), \( \text{Im} \varepsilon_i, \mu_i \leq 0 \). The boundary value scattering problem is well known to have an unique solution [4].

In the frame of Discrete Sources Method the approximate solution is constructed as a finite linear combination of the field of sources (dipoles and multipoles) deposited in a supplementary domain. Under these conditions the representation satisfies Maxwell equations everywhere and radiation conditions at infinity. The unknown amplitudes of discrete sources are to be determined from the transmission conditions at the particle boundary. So the boundary value scattering problem under investigation is reduced to the solution of an approximation problem enforced at an obstacle surface [5].

One of the most attractive features of DSM consists in flexible choice of DS fields that can be used for approximate solution construction. Additionally there are no limitations to a choice of support of DS, which should provide fulfilling Maxwell equations, radiation conditions and yield a complete system of DS fields at the obstacle surface [2]. For the oblate obstacles sometimes it's necessary to invent a special DS support. One of the possibilities DSM gives: to deposit DS in a complex plane. Such procedure allows us to limit of DS's sequence when the number of harmonics tends to infinity. The limitation is very important to provide the stability of numerical model based on DSM. The procedure of constructing of analytic continuation of DS's support to the complex plane one can find in [2]. We will consider an axial symmetric particle; in this case the system of lowest order multipoles distributed over the complex plane can be applied to construct an approximate solution. As a consequence the surface approximating problem can be reduced to a number of one dimensional problems enforced at the particle generator. The approximate solution will be constructed taking into account not only the rotational symmetry of the obstacle, but the polarization of an external excitation as well.

To take the polarization of the external excitation into account we use some linear combination of electrical and magnetic multipoles \( \{ w_n \}_{n=1}^{\infty} \) distributed over a complex plane. In case of P-polarization of the plane wave the representation for vector potentials in a cylindrical coordinate system can be represented as

\[
A_{m}^{1,\varepsilon,\gamma} = \{ Y_{m}^{\varepsilon,\gamma}(\eta, w_{n}^{\gamma,\varepsilon}) \cos(m+1)\phi; -Y_{m}^{\varepsilon,\gamma}(\eta, w_{n}^{\gamma,\varepsilon}) \sin(m+1)\phi; 0\},
\]
\[
A_{m}^{2,\varepsilon,\gamma} = \{ Y_{m}^{\varepsilon,\gamma}(\eta, w_{n}^{\gamma,\varepsilon}) \sin(m+1)\phi; Y_{m}^{\varepsilon,\gamma}(\eta, w_{n}^{\gamma,\varepsilon}) \cos(m+1)\phi; 0\}.
\]

The vector potentials for vertical dipoles, which are required to be added to provide completeness of the multipoles's system, are

\[
A_{m}^{3,\varepsilon,\gamma} = \{ 0; 0; Y_{0}^{\varepsilon,\gamma}(\eta, w_{n}^{\gamma,\varepsilon}) \}.
\]

Here: \( Y_{m}^{\varepsilon,\gamma}(\eta, w_{n}^{\gamma,\varepsilon}) = j_{m}^{\langle} \left( k_{\gamma} R_{\gamma} \right) \left( r/R_{\gamma} \right) m, \quad Y_{m}^{\varepsilon,\gamma}(\eta, w_{n}^{\gamma,\varepsilon}) = h_{m}^{(2)} \left( k_{\gamma} R_{\gamma} \right) \left( r/R_{\gamma} \right) m, \quad \)

where \( j_{m}^{\langle} \) is the spherical Bessel function, \( h_{m}^{(2)} \) is the spherical Hankel function, \( R_{\gamma} = \rho^2 + (z - \xi_{\gamma})^2, \quad \eta = (\rho, z) \in \Phi \) are the coordinates of the multipoles inside the particle, \( \Phi \) is a complex plane. So the approximate solution taking into account P-polarization of the plane wave and axial symmetry of the particle can be represented in the form.
\( \left( \begin{array}{c} E^N_w \\ H^N_w \end{array} \right) = \sum_{m=0}^{M} \sum_{n=0}^{N_n} \left\{ p^n_{mn} D_1 \times A_{mn}^{1\xi} + q^n_{mn} D_2 \times A_{mn}^{2\xi} \right\} + \sum_{n=0}^{N_n} r^n D_1 \times A_{n}^{1\xi}, \right) \quad (1) \\
D_1 = \begin{pmatrix} j \frac{1}{k^{\xi} \varepsilon^{\xi}} \nabla \times \nabla \\ \frac{1}{k^{\xi} \mu^{\xi}} \nabla \end{pmatrix} \\
D_2 = \begin{pmatrix} \frac{1}{k^{\xi} \varepsilon^{\xi}} \nabla \\ j \frac{1}{k^{\xi} \mu^{\xi}} \nabla \times \nabla \end{pmatrix} \)

To provide convergence of approximate solution to the exact one it is sufficient to provide the completeness of the system of distributed multipoles which are used for approximate solutions representation. The scattering from the S-polarized plane wave can be analyzed in the same manner [3].

Let us shortly describe the numerical algorithm. The approximate solution (1) satisfies Maxwell’s equations and radiating conditions at infinity. As the approximate solution satisfies all the conditions of the original scattering problem, except the boundary conditions, the unknown vector of amplitudes of discrete sources

\( p_m = \{ p_{mn}^{\xi}, q_{mn}^{\xi}, r_n^{\xi} \}_{n=1}^{N_n} \)

is to be determined from the boundary conditions at the particle surface. As the discrete sources are distributed on the symmetry axis of the particle, the approximate solution is a finite linear combination of Fourier harmonics with respect to \( \phi \) angle variable. The plane wave excitation can also be resolved into a Fourier series with respect to \( \phi \) angle variable. So, one can reduce the surface approximation problem enforced at the particle surface to a sequence of one-dimensional problems at the particle generator. To solve this problem the General Matching-Point Technique is used.

After discrete sources amplitudes \( \{ p_m \}_{m=1}^{M} \) have been determined, the far field pattern can be computed [6]:

\[
\frac{E(r)}{E^0(r)} = \frac{\exp\{-ikr\}}{r} F(\theta,\phi) + o(1/r), \quad r \to \infty.
\]

Using asymptotic representation for \( Y_{mn} \) for the exciting plane wave the components of the far field pattern can be calculated analytically [2].

We will mostly study the Differential Scattering crosssection (DSC)

\[ DSC = \left| F_{\varphi}^{P,S}(\theta,\phi) \right|^2 + \left| F_{\varphi}^{P,S}(\theta,\phi) \right|^2, \]

here \( F_{\varphi}^{P,S}(\theta,\phi) \) are the components of the far field pattern for P and S polarized excitation.

Another quite important characteristic of scattering is the integral response \( R \), which represents the flow of scattered energy through the unit sphere:

\[ R = \int_{\Omega} DSC \, d\omega, \]

where \( \Omega = \{ 0^\circ \leq \phi \leq 360^\circ; \ 0^\circ \leq \theta \leq 180^\circ \} \). Taking into account a possibility break into three: the integral over \( \phi \) into two integrals on \( \theta,\phi \), we will have a series of integrals which can be reduced to integrals of lower orders. The last integrals can be calculated analytically.
Results.

We will represent just some exemplary results calculated on the base of DSM. In all presented results we used a wavelength of $\lambda=488\text{nm}$, P-polarizes excitation and incident angle $\theta=150^\circ$, where $\theta=0$ corresponds to the axis of symmetry of the particle. The exact biconcave shape of erythrocyte was detailed discussed in [1], we will shortly mention just the main parameters: diameter $D=6.3\mu m$, length $l=1.575\mu m$ which for oblate objects is smaller then $D$ and $n=1.058$. The dependence of integral response from the incident angle (Fig. 1) and DSC from scattering angle (Fig. 2) for different shapes are presented. At Fig. 2 one can see that in range of scattering angle around $0^\circ$, which is mostly of interest in the experiment [1] the model of disk-sphere demonstrates perfect agreement with the rigorous biconcave erythrocyte model. More results will be presented at oral presentation.

Conclusion

The extended algorithm of DSM was applied to model light scattering by human erythrocyte. This became possible due to the special procedure which allows to deposite DS in the complex plane. That essentially helps to model such elongated objects, like spheroid, disk-sphere and biconcave erythrocyte. For comparison different models of erythrocyte together with a rigorous one were investigated. Obtained results allow us to make conclusion that the most acceptable model for modelling erythrocyte in the range of angles which is mostly important for experimental studies is disk-sphere and not the most common model of oblate spheroid if such approximation is still needed.

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REFERENCES