PEEC Equivalent Circuits for Dispersive Dielectrics

G. Antonini  
Department of Electrical Engineering  
University of L’Aquila  
Poggio di Roio, 67040 AQ, Italy  
e-mail: antonini@ing.univaq.it

A. Ruehli  
IBM Research Division  
Yorktown Heights, NY  
10598, USA  
e-mail: ruehli@us.ibm.com

A. Haridass  
IBM Server Division  
Austin, TX  
78758, USA  
e-mail: anandh@us.ibm.com

Abstract

Dielectric losses are becoming more important with the recent increase in the frequency content of signals propagating in high speed printed circuit boards. In this work, we provide a systematic approach for including losses in PEEC models. Recursive convolution and circuit synthesis techniques are employed to take into account the dispersive behavior of the dielectric. Presently, the algorithm has been applied to materials with a Lorentzian model for the complex permittivity. The accuracy of the proposed approach is demonstrated by numerical examples.

Introduction

The electrical properties of dielectrics vary with frequency. This phenomenon is known as dispersion and results into a convolutional relation between an impulsive excitation and the effective dielectric impulse response. The inclusion of dispersive effects into numerical 3-D full wave solvers is a challenging task. With the recent increase in the frequency content of signals propagating on printed-circuit board (PCB) transmission-line structures, dispersive behavior of dielectrics cannot be neglected anymore and their modelling has become of great concern. During the last decades the Partial Element Equivalent Circuit (PEEC) method and solvers have significantly evolved as more new features have been added to the approach. Finite dielectrics were added to solve a new class of multi dielectric problem [1]. In this work, we provide an approach which is applied to the dielectric model to include losses. It is evident that the inclusion of additional equations and especially circuit elements is particularly simple in a general stamp based circuit solver environment based on Modified Nodal Analysis (MNA) type techniques. For efficiency reasons, it may be desirable to include dielectrics in PEEC in several different ways. In [2] two different approaches are given for the inclusion of dielectrics. One is recursive convolution based [3] and the other is based on equivalent circuit synthesis which obviously is easily implemented in the PEEC circuit environment. The synthesized circuit model is another approach for the incorporation of losses into dielectric PEEC models. This approach has an advantage over the above time domain approach since it is a circuit which will be applicable to both the time and frequency domain without changes. In this paper we focus on the synthesis of equivalent circuits taking into account the dispersive behavior of the dielectrics. Single, double and multiple pole cases are considered and the corresponding MNA stamps are obtained. Furthermore it is shown that, being guaranteed the Hilbert consistency of the real and imaginary part for the dielectric material, the equivalent circuit are strictly passive. The frequency dependent permittivity of a dispersive dielectric can be approximated by a sum of Debye and Lorentz terms. For each of them an equivalent circuit is synthesized and incorporated in the PEEC solver. The proposed technique has been implemented in a full-wave 3D PEEC solver.

Basic Formulation for general dispersive media

Due to space constraints we assume that the formulation in PEEC for finite dielectrics and the circuit model are known [1,4]. The dielectric material is included with an excess capacitance which is given by:

$$C_e = \varepsilon_0 (\varepsilon_r - 1) \frac{S_m}{d_m}$$

(1)

where $S_m$ and $d_m$ denote the mean surface and thickness of the elementary dielectric cell volume. Here, we are interested in the case where the relative permittivity $\varepsilon_r$ is specified as a frequency dependent quantity. The electric flux is $D = \varepsilon_0 \varepsilon_r E$, where $\varepsilon_0$ is the permittivity of free space. In a
linear dispersive medium, the relation between the electric flux density and the electric field is given by

\[ D(t) = \varepsilon_0 \varepsilon_\infty E(t) + \varepsilon_0 \int_0^t E(t-\tau) \chi(\tau) d\tau \]  

(2)

\( \chi(\tau) \) is the electric susceptibility and \( \varepsilon_\infty \) is the infinite frequency permittivity where the convolution corresponds to a multiplication in the frequency domain. Assuming uniform time steps, for the time discretized form we get for the present time step \( t = t_n = n\Delta t \)

\[ D(t_n) \equiv D(n\Delta t) = \varepsilon_0 \varepsilon_\infty E(n\Delta t) + \varepsilon_0 \int_0^{n\Delta t} E(n\Delta t - \tau) \chi(\tau) d\tau \]  

(3)

Here, we use as an example the frequency dependent dielectric properties of a material generated from a physics based model. The electrical susceptibility \( \chi(t) \) from the model can be expanded as a strictly proper rational function in the Laplace domain as:

\[ \chi(s) = \frac{\beta_m s^m + \beta_{m-1} s^{m-1} + \ldots + \beta_0}{\alpha_p s^p + \alpha_{p-1} s^{p-1} + \ldots + \alpha_0} = \sum_{k=1}^{n} \frac{R_k}{s - s_k} \]  

(4)

where \( p > m \) and where the pole-residue representation on the right is found using a partial fraction expansion. Using the Laplace transform, the equivalent time domain representation is

\[ \chi(t) = \sum_{k=1}^{n} R_k e^{-s_k t} \]  

In [2] it is shown that, by applying such expansion to the PEEC model excess capacitance (1), then by inverse Laplace transforming the expression for the excess charge due to an applied voltage \( v_c(t) \) and finally by using a standard discretization technique we get a discrete recursive convolution scheme which reads

\[ q_c(n) = \varepsilon_0 (\varepsilon_\infty - 1) \frac{S_m}{d_m} v_c(n) + \varepsilon_0 \frac{S_m}{d_m} \eta(0) v_c(n) + \varepsilon_0 \frac{S_m}{d_m} \sum_{k=1}^{n-1} v_c(n-k) \eta(k) \]  

(5)

being

\[ \eta(0) = \int_0^{\Delta t} \left( \sum_{k=1}^{n} R_k e^{s_k \tau} \right) d\tau = \sum_{k=1}^{n} \frac{R_k}{s_k} (e^{s_k \Delta t} - 1) \]  

(6)

This recursive convolution approach is strictly for time domain PEEC solutions.

**Spice compatible circuit for dispersive dielectrics**

The synthesized circuit model is another approach for the incorporation of losses into dielectric PEEC models. This approach has an advantage over the above time domain approach since it is a circuit which will be applicable to both the time and frequency domain without changes. A frequency dependent permittivity can be approximated by a sum of Debye and Lorentz terms. Assuming \( N_D \) are the number of Debye terms and \( N_L \) the Lorentz terms, then we get the following representation of \( \varepsilon(s) \) in the Laplace domain

\[ \varepsilon(s) = \varepsilon_0 \left[ 1 + \sum_{m=1}^{N_D} \left( \frac{\varepsilon_{DS}(m) - \varepsilon_{DO}(m)}{1 + s\tau(m)} \right) + \sum_{m=1}^{N_L} \left( \frac{\varepsilon_{LS}(m) - \varepsilon_{LO}(m)}{s^2 + 2s\delta(m) + \omega_0^2(m)} \right) \right] \]  

(7)

where \( \varepsilon_\infty = \sum_{m=1}^{N_D} \varepsilon_{DO}(m) + \sum_{m=1}^{N_L} \varepsilon_{LO}(m) \), \( \varepsilon_{DS}(m) \) and \( \varepsilon_{LS}(m) \) are the static dielectric constants for the \( m \)-th Debye and Lorentz medium, \( \omega_0(m) \) is the resonance frequency of the \( m \)-th Lorentz medium. The loss constant of the \( m \)-th Debye term is \( \tau(m) \). The static permittivity of
such a medium can be defined as $\varepsilon_s = \sum_{m=1}^{N_d} \varepsilon_{DS}(m) + \sum_{m=1}^{N_l} \varepsilon_{LS}(m)$. The admittance corresponding to the PEEC excess capacitance can be represented as

$$sC_e(s) = sC_\infty + \sum_{m=1}^{N_d} Y_{DS}(s) + \sum_{m=1}^{N_l} Y_{LS}(s)$$

which can be synthesized into the circuit shown Fig. 1.

![Figure 1. Equivalent circuit for a general dispersive dielectric.](image)

Finally, this model can easily be incorporated into the PEEC MNA circuit solver since it consists of conventional circuit elements.

**MNA stamps**

The equivalent circuit shown in Fig. 1 is constituted by several Debye and Lorentz excess capacitance equivalent circuits in parallel. In order to develop the corresponding MNA stamp it is sufficient to provide that for single Debye and Lorentz terms. In the case of Debye media the stamp for the $C_{D\infty}$ capacity is straightforward. For the $RC$ branch the following equations reads:

$$v_c(t) = R_D i_{C_D}(t) + v_{C_D}(t) \quad i_{C_D}(t) = C_D \frac{dv_{C_D}(t)}{dt}$$

By applying a Backward Euler (BE) discretization scheme the following equations are obtained:

$$i_{C_D}(n) = G_{eq}(v_1(n) - v_3(n)) - i_{eq}$$

where $v_1$ and $v_3$ denotes the terminal nodes of the dielectric PEEC cell, $G_{eq} = \frac{C_D}{\alpha dt}$, $\alpha = 1 + \frac{R_D C_D}{dt}$, $i_{eq}$.

For Lorentz media again the stamp for the $C_{L\infty}$ capacity is straightforward. For the $RLC$ branch the following equations (omitting the subscript $RLC$ in the current) hold:

$$v_c(t) = R_L i_{C_L}(t) + L_L \frac{di_{C_L}(t)}{dt} + v_{C_L}(t) \quad i_{C_L}(t) = C_L \frac{dv_{C_L}(t)}{dt}$$
The use of the BE discretization scheme and some trivial algebraic manipulations lead to the MNA stamp:

\[
 i_{C_0}(n) = G_{eq}(v_1(n) - v_4(n)) - i_{s_{eq}}
\]

(12)

where \( G_{eq} = \frac{1}{dt/L + L_d/dt + R_L} \), \( i_{s_{eq}} = G_{eq}(v_{c_1}(n-1) - L_d/dt i_{c_2}(n-1)) \). The time discrete equivalent circuit synthesizing the Lorentz dispersive behavior for one PEEC cell is shown in Fig. 2.

![Image](image.png)

**Figure 2.** Time discrete equivalent circuit for a Lorentz medium.

**Numerical results**

In the first example we assume a simple Lorentzian medium with the parameters \( S_m = 0.05m^2 \) and \( d = 1mm \), filled with a medium whose permittivity is given by

![Image](image.png)

**Figure 2.** Voltage: left: input voltage; right: output charge.

where \( \varepsilon_{LS} = 4.32 \), \( \varepsilon_{Loo} = 4.12 \), \( f_0 = 15GHz \), \( \delta = 150GHz \). Here, we simply compute the excess capacitance which is driven by a pulse voltage \( v_c(t) \). The charge \( q_c(t) \) in response to the voltage source is shown in Figs. 2 where the output by using MNA stamps is compared with those obtained with the standard discrete convolution, the recursive convolution, the quasi static response and a state variable based on a built-in Matlab function (lsim). It is evident that the comparison is very good.

**REFERENCES**