New Artificial High-Permeability Material for Microwave Applications

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Abstract

Various possibilities to design artificial magnetic materials for microwave frequencies are considered. Such composites can be used in microwave engineering at frequencies where no natural low-loss magnetic materials are available. A new magnetic particle (metasolenoid) formed by a stack of many parallel single split-ring resonators is proposed and analyzed analytically, numerically, and experimentally.

1 Introduction

It is well-known that natural magnetic materials lose their magnetic properties at microwave and millimeter-wave frequencies. This means that at high frequencies magnetics have to be realized as composite materials with conducting inclusions of complex shapes. Various practical designs such as broken loops and pairs of broken loops [1, 2], the split-ring resonator (SRR) [3] and the so-called modified split-ring resonator (MSRR) [4], have been studied in the literature. The main drawback of all these designs is that the effective magnetism quickly vanishes when the frequency deviates from the particle resonance.

As a possible solution to this problem, we introduce a stack made from single SRRs. We call it metasolenoid. In this paper we show that it is possible to obtain high values of the effective permeability over a noticeably wider range of frequencies far away from the resonance as compared to the case of a SRR or a MSRR composites. Near the lowest resonant frequency all the mentioned structures can be described by means of the total effective current circulating in the inclusions. At low frequencies the magnetic moment is proportional to the effective capacitance. One of the goals of our design will be to increase this capacitance as much as possible.

2 Metasolenoids

2.1 Single metasolenoid

The geometry of the proposed artificial particle and used notations are shown in Figure 1. In the theoretical analysis we assume the structure to be infinite in the $z$ direction. The external magnetic field inducing currents on the surfaces of the rings is applied along the $z$ axis and is uniform inside the metasolenoid. The unit cell of the metasolenoid is the volume between two neighboring split rings (the shaded volume in Figure 1). The sum of the two currents which are internal to the shaded volume represents the effective current circulating in the unit cell. We denote it as $I$. 
The total magnetic flux in the metasolenoid can be expressed as
\[ \Phi = \mu_0 S H_{\text{ext}} + \frac{\mu_0 S J}{d} \]  
(1)

where \( S = a b \) is the cross-section area of the metasolenoid. The electromotive force produced by the total flux reads:

\[-j\omega\Phi = I \left( \frac{1}{j\omega C_{\text{eff}}} + R_{\text{eff}} \right) \]  
(2)

Utilizing the formula for the capacitance per unit length \( C_0 \) of a symmetric strip line [5], the effective capacitance can be expressed as:

\[ C_{\text{eff}} = \frac{C_0 l}{16} = \frac{\varepsilon_0 \varepsilon_r l}{4} \frac{K(k)}{K'(k)}, k = \left( \cosh \frac{\pi w}{2d} \right)^{-1} \]  
(3)

\( \varepsilon_r \) is the relative permittivity of the material which supports the rings and \( l = 2(a + b + g) \). The effective resistance \( R_{\text{eff}} \) is the sum of the radiation resistance and the loss resistance. Combining (1) and (2) we see that we can define the effective inductance of the metasolenoid as

\[ L_{\text{eff}} = \frac{\mu_0 S}{d} \]  
(4)

Finally, the total impedance of a metasolenoid is

\[ Z_{\text{tot}} = j\omega L_{\text{eff}} + \frac{1}{j\omega C_{\text{eff}}} + R_{\text{eff}} \]  
(5)

### 2.2 Material parameters

Let us estimate the effective permeability of a medium densely filled with many parallel metasolenoids. For the case of magnetic excitation \( H_{\text{ext}} = H_{\text{ext}} z_0 \) (see Figure 1) we can write for the magnetic flux density in the metasolenoid

\[ B_{\text{sol}} = \mu_0 H_{\text{ext}} + M_{\text{sol}} \]  
(6)

where \( M_{\text{sol}} = M_{\text{sol}} z_0 \) is the magnetic moment per unit volume of the metasolenoid. The amplitude of \( M_{\text{sol}} \) reads
\[ M_{sol} = \frac{\mu_0 J}{d} = -\frac{j\omega \mu_0^2 S H^\text{ext}}{Z_{tot} d} \]  

(7)

In a medium filled by infinite metasolenoids oriented along the \( z \) axis the average magnetic induction is

\[ \mathbf{B} = \mu_0 \mathbf{H}^\text{ext} + M_{sol} V_r \]

(8)

where \( V_r \) is the volume filling ratio. Finally, the effective relative permeability of the medium can be found as

\[ \mu_{eff} = 1 - V_r \frac{j\omega \mu_0 S}{Z_{tot} d} \]

(9)

In the measurements we will use a finite size segment of a metasolenoid. The magnetic polarizability of the sample depends on its shape. To take into account this influence we approximate the sample shape by an ellipsoid. Utilizing the known formula for the polarizability of the ellipsoid (e.g. [6]) we can write for the magnetic polarizability of the metasolenoid

\[ \alpha_{\text{mm}} = \frac{\mu_0 \pi}{6} \frac{abh(\mu_{eff} - 1)}{1 + N_c(\mu_{eff} - 1)} \]

(10)

where \( h \) is the longitudinal length of the metasolenoid and \( N_c \) is the depolarization factor along the longitudinal axis.

3 Analytical results and experimental verification

As an example we analyze a metasolenoid with the parameters given in Table 1. The notations refer to Figure 1. The loss tangent of the laminate supporting the rings equals tan\( \delta = 0.002 \).

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>( w )</th>
<th>( g )</th>
<th>( d )</th>
<th>( \varepsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>3.5</td>
<td>0.4</td>
<td>1.0</td>
<td>0.127</td>
<td>2.2</td>
</tr>
</tbody>
</table>

| Table 1: The parameters of the metasolenoid sample |

Figure 2 shows the analytically calculated dispersion curve for the polarizability and the relative permeability of the metasolenoid in the case when a medium is densely filled with metasolenoids, such that volume fraction approaches unity.

![Figure 2: The polarizability \( \alpha_{\text{mm}} \) (top) and the permeability \( \mu_{\text{eff}} \) (bottom): analytical model.](image)
A standard one-meter long WR-650 waveguide with two transitions to coaxial cables has been used in the measurements. The overall dimensions of a metasolenoid segment used in the reflection measurements are $6.5 \times 6.5 \times 7.6 \text{ mm}^3$. Because of a small size compared to the wavelength we approximate the metasolenoid section as a magnetic dipole and extract the polarizability from the measured results utilizing the known theory for waveguide excitation [5].

The comparison between the analytical and measured results is shown in Table 2. The measured values agree qualitatively well with the theoretical predictions. The resonant frequency predicted by HFSS simulations is presented in brackets.

|                     | $f_{res}$ | Re{$\alpha_{\text{max}}$} | $|\alpha_{\text{max}}|$ | Re{$\mu_{\text{eff}}^{\text{max}}$} |
|---------------------|----------|--------------------------|------------------------|---------------------------|
| Theory and simulations | 0.998    | $1.7 \times 10^{-11}$    | $3.3 \times 10^{-11}$  | 230                       |
| Measurements        | 1.21     | $1.4 \times 10^{-11}$    | $2.7 \times 10^{-11}$  | -                         |

Table 2: Theoretically predicted and measured parameters of the metasolenoid sample

4 Conclusion

A simple analytical model for a new artificial magnetic inclusion has been introduced and validated by simulations and measurements. We can point out that the real part of $\mu_{\text{eff}}$ remains high over a very wide frequency range (Figure 2) exceeding 10 still at frequencies about five percents lower than the resonant frequency of the metasolenoid.

REFERENCES


