Homogeneous Magnet Design Using Genetic Algorithms

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Abstract
Authors introduce a technique for designing homogeneous magnet using Genetic Algorithms (GA). In the paper authors present the historical approach to the problem and farther the advantage of using GA presenting the example calculation.

1. Introduction

Building magnets exciting the homogeneous magnetic field is the problem known for many years. Then problem of the exciting homogeneous field was linked with the building of the advanced medical constructions like magnetic resonance imaging (MRI), but also the problem can be seen in the as well magnetic filtration [1] as in the bubble chamber magnets for particle detections or magnets for chemical analyses by nuclear magnetic resonance (NMR). As well in MRI as in magnetic filters the excitation of the large value of the magnetic field in the large volume is required. Such field usually is created by superconducting coils. In the process of the coil construction one has to take into consideration characteristic of the superconducting material.

Figure 1 presents the magnet design problem. The feasible coil space is densely sampled by an array of candidate coils. The coils are assumed to be ideal current loops located at the center of the squares. The goal is to pick the right coils and currents values to minimize (maximize – depends on the aim function construction) value which compare the field created with the field required at the target points, in our case a most homogenous (constant). The shape of homogeneous volume is entirely arbitrary. Each discrete current loop is expressed by its current \( I_i \), radius \( r_i \) and its position \( z_i \).

Each coil generates a magnetic field contribution at each target point. Comparing figures 2 one can calculate this contribution from one simply coils using the equation:

\[
B_j(r_j, z_j) = \frac{\mu I_i}{2\pi\left((r_j + r_i)^2 + (z_j - z_i)^2\right)^{3/2}} \left( E(\kappa) + K(\kappa) \right)
\]

where \( B_j \) is \( z \) component of the magnetic field generated at the \( j \)-th target point due to current \( I_i \), and \( E(\kappa) \) and \( K(\kappa) \) are complete elliptic integrals of the first kind and second kind accordingly, and the factor \( \kappa \) is expressed by:

\[
\kappa = \frac{4r_i r_j}{\left(r_i + r_j\right)^2 + (z_i - z_j)^2}
\]
2. Methods review

The field $B_z$ from $n$ such coils is simply the sum of the magnetic field generated by all of them. This is a usage of the superposition principle which can be applied only in linear medium.

Many approaches to the problem mentioned above were done. Starting from analytical approach, yet in 50’s first works appeared, especially the classical papers of McKeehan, Garrett, and the others [2]. In 1986, Lugansky used nonlinear optimization to find a set of N coils that minimized the mean-squared field error [3]. In 1992, Pissanetzky introduced a method that minimized the mean-squared field error subject to hard constraints on the magnitude of the currents [4]. This method is very powerful, but the cost of such real design could be great due to the field cancellation from opposing currents.

There are also several methods using linear matrix approaches to solve magnet design problem. These algorithms are very efficient and in theory should always find the global solution. However, there is a problem with finding the practical solution which could be applicable in the real objects. These methods attempt to invert the matrix relating the field $B$ at the $m$ target points with the currents in the $n$ feasible coils.

$$B = A I$$

where: $A_{mn}$ is the field per unit current at $m$-th target point due to the unit current in the $n$-th feasible coil, $B$ is the column matrix describing the magnetic field at target points and $I$ is the column matrix of source currents. Direct methods of inverting the matrix $A$ are useless due to $A$ is very poor conditioned. Some authors solved this problem using Fourier domain method. But this approach gives very complex and very non-homogeneous current distribution in the coil. In 80’s interesting approach was performed by Palka and Sikora [5]. They stated that such problems are the ill-posed, and method of regularization of matrix $A$ is needed. In this case Tikhonov regularization was applied.

3. Homogenous magnet design using Genetic Algorithms

So far the solutions are based on the deterministic algorithms. However there is the big field of the non deterministic (stochastic) genetic algorithms (GA) and its generalization – the evolutionary computation. They are very “flexible”, simple and easy to apply. Unlike the deterministic solutions GA operates on the set of the individuals. Each individual represents the possible solution of the problem. The set is called the population. Each individual is assessed by the value which presets its adaptation. These values allow distinguish which individual is closer or farer to the solution. Operation of the GA is reduced to the loop presented on the figure 3. In the loop some genetic operation are performed in turn (reproduction, selection, mutation and crossover). This loop should be repeated until the best solution (global) is found. It is worth to add here that in theory the best solution will be found when the amount of repeat of the loop is infinity. In other words the probability of achieving the best solution is greater when the number of the performed loop is greater. There is many criterions that allows stopping the algorithm, they has been in exhaustive way described in [6].

![Figure 3. Loop of the simple GA](image)

In the GA very important aspect is coding, i.e. the way the individual is represented in the algorithm. Authors coded the real coil using the binary system with the Gray code.
4. Examples and results

Authors done the simulation on the simple situation when the coil is divided into 1 x n subsections (figure 4). For each subsection algorithm has to find the current value which minimize the homogeneity factor \( \varepsilon = \sqrt{\int \left( B_z - B_{req} \right)^2 \, dz / B_{req}} \). Where \( B_z \) is the field value in the z axis and \( B_{req} \) is the field required, in our case the constant value (homogeneity). In other words algorithm has to find the current distribution in the coil that excites the most homogenous field. At the starting point algorithm has to generate the beginning population. In our case it will be the number of coils with the randomly assigned the current value to each subsection. The value of the current must fulfill the maximum value of current density in each subsection for the used material. On figure 5a there is presented the exemplary individual form the beginning population and responded field distribution (figure 5b). Then from the population there were chosen the individuals for the genetic operations. Selection has been done using the roulette method i.e. probability of selecting the individual to further genetic operators is proportioned to the better adaptation (lower \( \varepsilon \) value).

Selected individuals was undergone the crossover operation. Authors introduced two kinds of this operator: indexed and real (figure 6). Selected individual has been undergone the mutation operation. Mutation and crossover has been done with given probability (\( p_{cross}=0.7; \ p_{mut}=0.05 \)).

Effect of the algorithm work is presented on figure 7. Generations has been stopped right after the 2000 loops. Here it is worth to add that the individual founded after 2000 generations is not the exact the global optimal. One can see the little lack of symmetry on the current distribution.
On figure 8 authors present the curves of convergence for the algorithm. Two values could be seen, the $\varepsilon$ value of the best individual for the generation and average $\varepsilon$ value of the whole population. Algorithm was done with the following parameters: population size – 75; number of the maximum generation – 2000; $p_{cross}$ – crossover probability – 0.8; $p_{mut}$ – probability of mutation – 0.08; crossover method – indexed.

![Figure 8. Convergence curves for the example calculations](image)

Further authors considered the situation when the coil is divided into $n \times m$ subsections. Figure 9 presents the result of calculated current distribution in the coil that minimizes homogeneity factor $\varepsilon$. This example is calculating for the superconducting coil for the High Gradient Separator [1] where high field value (4-5T) is required. In this case the mean value of the field on the $z$ axis (-20÷20cm) is equal 4.85 T ($B_{req}$=5 T) with $\varepsilon$ factor minimized to 0.098. Coil has dimension 3.5cm x 32cm. Coil was divided into 21x43 elementary coils loops. The evaluation algorithm was carried out with the population of 65 individuals, probability of crossover was 0.8, mutation 0.1. Condition of the evaluation stop was fulfilled after 1187 generations. In this case the condition was that for the best individual the $\varepsilon$ factor is not changed in 50 generations.

5. Conclusion

The important problem of shaping the magnetic field distribution in the desired region could be solved with many different methods. The short review has been presented in the paper. Modern tool that could be used are Genetic Algorithms. It is the “flexible” and what is more important effective tool what is pointed out in the paper by the presented results gained for the exemplary coils configurations. Effect of the GA action is presented on figure 9. Figure shows the current distribution in the multilayer winding guaranteeing the assumed magnetic field homogeneity.

REFERENCES

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