Adaptive Transient Simulation of Field-Circuit Coupled Problems Including Switching Circuit Elements

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Abstract

The coupling between a 3D modified magnetic vector potential formulation discretized by the Finite Integration Technique and an electrical circuit containing switching elements such as diodes is presented. The changes in the state of the switching elements ("conducting" or "blocking") are introduced as changes in the topology of the circuit part of the model which results in a structural change of the differential-algebraic system of index 1 that describes the coupled field-circuit problem. The presented paper treats this topological aspect as well as the application of the classical and higher-order time integration methods, which allow for an elegant implementation of the adaptive time-stepping procedure.

Introduction

For the magnetodynamic simulation of electric energy converters, discrete models derived with geometric discretization methods such as the Whitney Finite Element Method or the Finite Integration Technique (FIT) allow to accurately consider local effects such as nonlinear ferromagnetic material behavior and eddy currents [1],[2]. The excitation of the discrete magnetodynamic system often arises from an electrical network which is introduced by lumped elements and can be modeled in terms of the Kirchoff equations.

The FIT discretization of the magnetodynamic part of the model and its coupling with external electrical circuits result in large differential algebraic systems of equations of index 1. If the external circuit part does not contain any switching elements, like e.g. diodes, thyristors and transistors, the structure of the system does not change during the simulation. A common approach to model switching circuit elements is to represent them by nonlinear resistivities with

$$R(t) = \begin{cases} R & \text{if } \text{diode is conducting} \\ \infty & \text{if } \text{diode is blocking} \end{cases}$$

[3], thus also allowing to keep the system unchanged. Since this approach results in very large condition numbers of the algebraic systems to be solved, here an alternative representation is chosen related to transient changes in the topological configuration.

Discrete Field-Circuit Coupled Formulation

The 3D transient field-circuit coupled formulation discretized by FIT ([2],[4]) reads

$$\begin{bmatrix} M_v & 0 & 0 \\ -Q^\top M_v & -Q^\top C & -P^\top & \mathbf{1} \\ -P & 0 & -N_v & 0 \end{bmatrix} \begin{bmatrix} \dot{a} \\ \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{C}} M_v C & -M_v Q^\top & -P^\top & \mathbf{1} \\ -Q M_v & 0 & 0 & N_v \\ -P & 0 & -N_v & 0 \end{bmatrix} \begin{bmatrix} a \\ v \\ i \end{bmatrix} = -D^{(s)} v_s + T^{(s)}. \quad (1)$$

The part of (1) corresponding to the field problem contains the path integrated magnetic vector potentials $\mathbf{a}(t)$, the positive semi-definite curl-curl stiffness matrix $\tilde{\mathbf{C}} M_v C$ and the conductivity matrix $M_v$ which is commonly singular [1].

The circuit formulation is based on a decomposition of the circuit graph into a tree and a cotree favoring independent voltage sources, solid conductors and capacitors be the tree branches. The prescribed voltages and currents in the independent sources appear at the right-hand side as the vectors $v_s$ and $i_s$, respectively.
For the remaining dependent circuit elements, the chosen state variables are voltage drops $v$ along the tree branches and currents $i$ in the cotree branches. Kirchoff’s current and voltage laws are expressed for the fundamental cutsets and loops. The incidences of the fundamental cutsets associated with the dependent tree branches with respect to the dependent cotree branches are collected in matrix $D^{(s)}$.

The lumped elements are gathered in the conductance matrix $N^{(s)}_g$, resistance matrix $N^{(s)}_r$, capacitance matrix $N^{(s)}_c$, and inductance matrix $N^{(s)}_l$.

If the circuit part of the model is to be subjected to partial cutest/loop transformation, the terms $s^Tv$ and $s^Ti$, containing the contributions from the voltage sources acting like tree branches and from the current sources acting like cotree branches together with the corresponding Schur-complements for $N^{(s)}_g$, $N^{(s)}_r$, $N^{(s)}_c$, and $N^{(s)}_l$, appear in the right-hand side of the coupled formulation.

The upper index $(s)$ prescribed to some blocks indicates that these blocks are subjected to changes due to the topological transformation of the system. For particular circuits, additional circuit contractions are required to achieve the prescribed circuit formulation.

The detailed description for constructing coupling blocks $M_z Q$ and $P$ is provided in [5].

**Modeling of Ideal Switching Elements**

Switching elements such as e.g. diodes are commonly modeled by non-linear resistors for which the conducting and non-conducting state are represented by very small (e.g. $10^{-12}$) and very large (e.g. $10^{12}$) resistance respectively [3]. This approach leads to the very big condition number of the linear systems to be solved and, as a consequence, to the necessity to find a good preconditioning for the system matrix. An alternative treatment is to change the topology of the circuit part with respect to the diode states, e.g. by replacing a diode with a zero voltage source for the conducting state and with a zero current source for the blocking state. The switching time instant has to be determined. Exactly at these switching instants, the time integration process has to be stopped and restarted again with a modified system of form (1) and with new start values for the differential-algebraic system corresponding to the new circuit topology. Apart from the reasonable condition numbers of the so far obtained systems, this approach is universal for further modeling of real physical diodes. For example, a Zener diode can be modeled as parallel connection of two ideal diodes where the first is in series with Zener voltage element, and the second is in series with forward cut-in voltage element. The desired values for the reciprocals of slopes in breakdown and in forward-biased regions are introduced now just like additional resistivities in the circuit part.

**Adaptive Time Integration**

Formulation (1) represents an ill-conditioned index 1 differential-algebraic system of equation (DAE) of the form

$$\frac{d}{dt} x = K x = r(t). \quad (2)$$

The application of the $\theta$– time discretization scheme and the implicit Runge-Kutta methods to this type of systems requires the solution of large indefinite linear systems which are then solved by eliminating the circuit degrees of freedom and constructing a Schur-complement to the FIT part of the system matrix [2]. As numerical tests in [2] and [6] show, some $\theta$– methods exhibit nonphysical oscillations for this type of problem and only suitable DAE integrators should be used. Therefore, we concentrated on only higher-order integration methods, namely singly diagonally implicit Runge-Kutta (RK) methods [2]. The coefficients of the corresponding Butcher table can be found in [7].

For each integration step, stiffly accurate embedded implicit RK methods deliver a solution of the given order $p$ and an embedded solution of a lower order $\hat{p}$. Such embedded pairs use the same matrix of the Butcher table but different update weight vectors. The difference between the two
solution builds an error vector $\tilde{y} = y^{(p)} - x^{(p)}$ a suitable norm for which should be found to initiate the time-stepping process.

**Time-Step Selection Procedure**

For the numerical experiments described in the next section the following norm for the error vector was used:

$$\left\| \tilde{y} \right\|_{\text{err}} = \max_i \frac{\left| y_i \right|}{\left| x_i + \delta \right|},$$

(3)

where $x$ is the solution of higher order and $\delta$ denotes an absolute tolerance for this solution.

The choice whether the last integration step has to be repeated or a new simulation step can be performed, is dictated by a step-size controller. According to [8], an I-controller is implemented by

$$\Delta t_{n+1} = \rho \left( \frac{\text{tol}}{\left\| \tilde{y} \right\|_{\text{err}}} \right)^{1/p} \Delta t_n$$

(4)

and the values for the parameters and settings for this procedure can be found in [9].

For the field-circuit coupled formulation with switching elements this standard time-stepping procedure was modified. Namely, after the I-controller determined a new time-step and the system was integrated numerically using this step, a checking routine is applied to determine if switching elements should have changed their state the time interval. As soon as one switching action detected, the predicted time-step length is reduced in order to stop in the first switching time instant. Afterwards, the topology is changed and the integration is restarted with a modified system. Till the next switching point the integration process and the adaptive time stepping are implemented in a standard way. The change of the topology implies also that the number of the degrees of freedom and the tree/cotree partitioning of the circuit graph changes.

**Numerical Tests**

The used test model consists of a coil which can be modeled by a stranded conductor ([2]), connected to the electrical circuit with two switching elements and two resistors (Fig.1a). The model is excited by a transient voltage source with the function $V_{\text{src}} = (230V)\sqrt{2}\sin(\omega t)$ (Fig.1b). For this test model, the coupled formulation of form (1) was derived and then treated with singly diagonally implicit four-stage RK method of order three with the embedded solution of order two (SDIRK3(2)). The sampling of the voltage curve in the first diode is presented in Fig.2. Time stepping markers on the second curve allow to conclude that from that time instants when the switching in the system was detected, the I-controller determines rather small initial steps increasing them consequently till the next switching moment.

**Conclusion**

Implicit RK time-integration methods with embedded solutions of higher and lower order were applied to simulate the behavior of a coupled field-circuit system with switching elements. The nonlinear behavior of the diodes causes the necessity to modify the standard time-stepping I-controller, namely to stop integration at switching points and to restart the transient simulation after modifying the topology of the circuit part. This approach also allows to keep the condition numbers of the obtained systems rather small.

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Figure 1. (a): test electrical circuit model with stranded conductor and switching elements; (b): excitation curve;

Figure 2. Adaptive time sampling of the voltage curve in the first diode

REFERENCES