Influence of Conductor Segmentation in Grounding Resistance Calculation Using Boundary Element Method

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Abstract

This paper presents different models of conductor segmentations in grounding resistance calculation of simple geometry (finite straight rod). Grounding resistance is calculated using Boundary Element Method (BEM) and results are compared with results obtained by analytical equation and professional CDEGS (Current Distribution Electromagnetic Interference Grounding and Soils Structure Analysis) software, which use Moment Method. Simple algorithms and programs are developed for calculating grounding resistance using integral equation method. Examples and results related to the several different types of conductor division into segments (rings and sticks) are presented for grounding resistance and surface leakage current density. The straight rod is a 1m-length conductor with circular cross section 1 cm radius, deeply buried in soil conductivity 0.01 S/m. The accuracy of different segmentation types is compared with low frequency module of well-known CDEGS software. Summarized results of grounding resistance for different approaches and analytical equation are presented.

Key words: Grounding resistance, numerical method, Boundary Element Method, simple grounding system, leakage current distribution.

Introduction

Because all numerical methods work on the basis of intuitive ideas such as: error averaging, superposition of punctual or another type of source, segmentation of conductor, many problems can be expected from physical to mathematical nature. In this paper is used division of conductor into sticks and rings with different types of conductor surface current interpretation: punctual and distributed, and is investigated numerical behaviour of each model. Grounding conductor is a simple finite rod without round at its ends. Conductor is buried into uniform infinite medium, so the influence of boundary between air and soil can be neglected.

Division of conductor into sticks – model with punctual sources

In order to derive expressions for numerical calculation, conductor is segmented into N horizontal segments as shown in Fig. 1. In this model the discrete (punctual) source is placed in the middle of each segment.

Figure 1. Position of observation point and sources with relevant geometry

According to the arbitrary indexing of punctual sources expressed in linear leakage current density $\lambda_i$ for the purpose of comparison with another models, first current densities have index “0” and the last one have index “N-1” (see Fig. 1). The observation profile is chosen along the surface of the rod. Position of each current density $\lambda_i$ and observation point $r_l$ in global coordination system is given as: $r_l = (-l/2 + \Delta \cdot (i + 1/2)) \hat{x}_i + b \cdot \hat{x}_2$ and $\bar{r}_l = (-l/2 + \Delta \cdot (j + 1/2)) \hat{x}_i + 0 \cdot \hat{x}_2$. Scalar potential for each observation point (middle of each segment), is given by equation:
\[
\varphi_i = \frac{\Delta}{4\pi \kappa} \sum_{j=0}^{N-1} \frac{\lambda_j}{\sqrt{(\Delta(i-j))^2 + b^2}} = \frac{\Delta}{4\pi \cdot \kappa \cdot N} \sum_{j=0}^{N-1} \frac{\lambda_j}{\sqrt{(\Delta(i-j))^2 + b^2}}
\]  

(1)

Where: \(\kappa\) - soil conductivity [S/m], \(\Delta\) - segment length (\(\Delta=l/N\)), \(b\) - conductor radius [m], \(\lambda_j\) - leakage linear current density on j-th segment [A/m]. Equation (1) generates a system of equations [\(\varphi_i = [p][\lambda_j]\)] that can be solved with one of numerical method (in this paper LU-decomposition is used).

Division of conductor into sticks – model with distributed sources

Another solution of the given modelling problem consists of using analytical equation for the potential around finite straight wire (\(b << l\)). Analytical equation for scalar potential around finite straight wire is given by equation (2), which is related to Fig. 1.

\[
\varphi(x_1, x_2) = \frac{\lambda}{4\pi \kappa} \ln \left( \frac{x_1 + c + \sqrt{(x_1 + c)^2 + x_2^2}}{x_1 - c + \sqrt{(x_1 - c)^2 + x_2^2}} \right)
\]  

(2)

Equation (2) is derived under assumption that surface current density can be described with linear leakage current density (distributed sources). Dividing the conductor into segments previously described, equation (2) describes the potential around each segment correctly, because current density can be assumed as constant on very small segment. In that case scalar potential is given by equation:

\[
\varphi = \sum_{j=0}^{N-1} \frac{\lambda_j}{4\pi \kappa} \ln \left( \frac{\Delta(i-j) + \Delta/2 + \sqrt{(\Delta(i-j) + \Delta/2)^2 + b^2}}{\Delta(i-j) - \Delta/2 + \sqrt{(\Delta(i-j) - \Delta/2)^2 + b^2}} \right)
\]  

(3)

Equation (3) generates a system of equation with N unknowns previously described.

Division of conductor into rings – model with discrete and distributed sources

Segmentation into rings is often used when highly accurate calculation is performed. Segmentation procedure is similar to the previously described. When one thin ring is placed around middle of segment \(\Delta\) it is appropriate to talk about discrete (quasi discrete) sources interpretation, but still better than equation (1), even current is distributed along ring radius, it is not distributed along segment length \(\Delta\). A ring segmented conductor is presented in Fig. 2.

After segmentation into desired number of segments (N), scalar potential of each observation point due to current density along ring \(\lambda'\) is obtained, using the equation for thin ring:

\[
\varphi(r,z) = \frac{\lambda' \cdot r'}{\kappa \pi \sqrt{a+b}} \int_0^{\pi/2} d\Phi \frac{d\Phi}{\sqrt{1 - m^2 \sin^2 \Phi}} = \frac{\lambda' \cdot r'}{\kappa \pi \sqrt{a+b}} K(\pi/2, m)
\]  

(4)

Where: \(a = r^2 + r'^2 + (z - z')^2\), \(b = 2 \cdot r \cdot r'\), \(z' = z\), \(m = \sqrt{2b/(a+b)}\). Symbol \(K(\pi/2, m)\) denotes complete elliptic integral of the first kind. Precautions should be taken since \(K(\pi/2, m)\) has singularity when \(m \to 1\), which is in the case when the potential of thin ring is obtained due to self-current. However, the divergence is only logarithmic and solution can be obtained using approximate equation \(\lim_{m \to 1} K(\pi/2, m) = \ln(4/\sqrt{1-m^2})\). By defining ring minor radius \(R_0 << R\), which can be
arbitrary small, for instance \( R_0 = \Delta/10 \) when \( \Delta < R \) for any observation point on the ring surface with coordinates: \( r = R, \ r' = R, \ z = R_0, \ z' = 0 \) we get \( 1-m^2 = (R_0 / 2R)^2 \). Thus critically diagonals elements \( p_{\text{self}} = p_{11} \) and non-diagonal elements \( p_{i,j} \) \((i \neq j)\) are given as:

\[
p_{\text{self}} = p_{11} = \lim_{m \to \text{w}} \left[ \frac{1}{K} 2 \left( \frac{\pi}{2}, m \right) \right] = \frac{1}{K} 2 \ln \left( \frac{8R}{R_0} \right). \quad p_{i,j(i \neq j)} = \frac{R \cdot K(m_{ij})}{\pi \cdot \kappa \sqrt{a_{ij} + b_{ij}}} \quad (5)
\]

Integral singularity can be avoided using double integration technique. In that case each segment is additionally subdivided into rings with equal linear current density along rings \( \lambda' \), so that surface current are interpreted as distributed current.

**Numerical Results**

Numerical calculation is performed using MathCAD 2000 software and professional CDEGS software. The rod is maintained at a constant potential of 1 V.
Figure 7. Current distribution. Results obtained using low frequency model in CDEGS software (N=25)

Summarized result for grounding resistance using the described approaches, analytical equation and professional CDEGS software is presented in Table 1. The analytical equation (2) gives as the value of 73.332 Ω for grounding resistance.

<table>
<thead>
<tr>
<th>Type of segmentation/ source interpretation</th>
<th>Grounding resistance [Ω] / number of segments [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sticks/ equivalent punctual currents Eq. (1)</td>
<td>75.377 68.426 67.663 73.417 68.119 63.876</td>
</tr>
<tr>
<td>Sticks/ distributed currents Eq. (3)</td>
<td>67.026 66.331 67.146</td>
</tr>
<tr>
<td>Rings/ equivalent discrete currents* (R₀=Δ/10)</td>
<td>73.417 70.099 68.607 67.878 67.510 67.226</td>
</tr>
<tr>
<td>Rings/ equivalent discrete currents* (R₀=Δ/6)</td>
<td>68.119 67.504 67.112 67.124 67.124</td>
</tr>
<tr>
<td>Rings/ equivalent discrete currents* (R₀=Δ/4)</td>
<td>63.876 65.240 66.121 66.504 67.055</td>
</tr>
<tr>
<td>Rings/ distributed currents/ double integration</td>
<td>67.026 67.095 66.121 67.124 67.124</td>
</tr>
<tr>
<td>CDEGS software/ Moment Method</td>
<td>67.692 67.331 67.146</td>
</tr>
</tbody>
</table>

* (1) - abnormal current density, * (2) - program restriction (no thin wire theory), * (3) - quasi discrete

Conclusion

Computational results indicate that for large number of segments in case of using equation (1), numerical results for line current density give non-physical solution near the end of rod. It can be seen from Fig. 3 and Fig. 4, when the number of segments is increasing, more accurate distribution of current is obtained until segments lengths Δ become of less than two times radius. In that case leakage current density near the ends of conductor exhibits unexpected drops (see Fig 4), which of course does not have physical background, although being mathematically correct solution. Results for current distributions using equation (3) are very similar to results using equation (1). The reason lays in the fact that for sufficient number of segments equation (1) practically describes potential of distributed sources, which is inherent feature of the equation (3). Only difference between equation (1) and equation (3) are in convergence (see Table 1.). From Fig. 5, we can see, that using the elliptic integral provides correct both mathematical and physical solution of the problem, even for the large number of segments. Results obtained from elliptic integral with double integration technique can be the reference both for grounding resistance and current distribution. As we can see from Table 1, professional software CDEGS gives R=67.146 Ω as result for grounding resistance, that is in good agreement with result obtained using elliptic integral. Since CDEGS practically used equation (3) it is obviously that convergences of results for grounding resistance, see Table 1., and current distributions are practically identical with results obtained with equation (3) see Fig. 6. and Fig. 7. As the results suggest all models with distributed currents give much better accuracy for smaller number of segments, but converge slowly compared to models with discrete (punctual) interpretation of sources.

REFERENCES