On Using Finite-Differences in Optical Pulse Propagation Modeling

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Abstract

In the paper we describe possible numerical models for modeling optical link components. Using the criteria of best accuracy and least computational time needed we have chosen finite-differences for optical fiber modeling. We also analyzed the use of various boundary conditions in numerical model and concluded that transparent boundary conditions (TBC) take less computational time than absorbing boundary conditions (ABC). Several simulation examples are given to illustrate the abilities of the simulation program FiberProp we developed.

INTRODUCTION

In the research of modern optical communication systems it became common to use computer simulations even before the field trials and commercial usage of systems. The reason for that lays in quick development of the field, i.e. short time passes between research and inventions in the lab and commercial use of them, as well as in expensive equipment needed for the research experiments. Therefore, there is a large interest worldwide in programs that can analyze all relevant components and effects in optical communication systems.

We developed computer program FiberProp, which consists of various fiber models i.e. pulse propagation models, models of optical amplifiers and optical filters in order to simulate the whole optical link. The critical among these components is the fiber model because pulse propagation in optical fiber is governed by nonlinear Schrödinger differential equation (NLSE), which can be numerically modeled on various ways [1,2]. We modeled it using finite difference method (FDM) and split-step Fourier method (SSFM). The accuracy test of both methods gave the FDM as more accurate one.

When one uses the finite differences, special attention is always given to initial values and boundary conditions [3]. In the program we developed, one can chose between various initial pulse shapes in sequences of up to 16 bits [4,5]. For the canonical test cases zero boundary conditions are used. They are the simplest boundary conditions and are easy to implement. For the more complicated cases of pulse sequences propagation along the optical links consisting of real fiber (with losses), erbium-doped fiber amplifiers and optical filters, one has to use the absorbing (ABC) or transparent (TBC) boundary conditions.

NUMERICAL MODELS

Optical pulse propagation in optical fiber is governed by partial differential equation, so-called NLSE in a form

\[
\frac{\partial A}{\partial z} = -\beta_1 \frac{\partial A}{\partial t} - j \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{\alpha}{2} A + j \gamma |A|^2 A
\]

where \( A \) is the pulse’s envelope. The first term on the right side describes the group velocity of the pulse \( \beta_1 = 1/v_g \), the second one represents the group velocity dispersion and the last two ones contain the influence of the fiber loss (coefficient \( \alpha \) [km\(^{-1}\)]) and the nonlinear effect of self-phase modulation (taken into account through the nonlinear coefficient \( \gamma \) [W\(^{-1}\)km\(^{-1}\)]). In order to numerically model the pulse propagation it is suitable to force the computational window to move with the whole pulse at group velocity i.e. to apply first the so-called ‘moving frame transformation’ on the NLSE. The transformation sets the new time variable \( T = t - z \beta_1 = t - z/v_g \). After employing the moving frame
transformation on (1) we obtain
\[
\frac{\partial A}{\partial z} + \frac{\beta}{2} \frac{\partial^2 A}{\partial T^2} + \frac{\alpha}{2} A = j\gamma |A|^2 A
\]

(2)

The equation (2) can be now easily split into two coupled equations (one describing the GVD and the other describing the nonlinear effect and fiber losses) which are then separately numerically modeled. This set of coupled equations is
\[
\begin{align*}
\frac{1}{2} \frac{\partial A}{\partial z} &= -j\beta \frac{\partial^2 A}{\partial T^2} \\
\frac{1}{2} \frac{\partial A}{\partial z} &= j\gamma |A|^2 A - \frac{\alpha}{2} A
\end{align*}
\]

(3)

In other words, we divide fiber into a set of concatenated \(\Delta z\) long segments. At each second segment the influence of dispersion on the optical pulse is considered, and at each remaining segment the impact of fiber nonlinearity and fiber loss is taken into account. Now, the proper method (FDM or SSFM) can be applied to model (3) according to the prescribed initial and boundary conditions. If \(\Delta z\) is small, one can suppose that \(|A|\) is not changed along the segment and the analytical approximate solution of the second equation can be obtained. The equation describing the influence of dispersion in the fiber is modeled using the Crank-Nicholson scheme as
\[
\begin{align*}
\Psi_{i+1/2}^{n+1/2} + j\beta_{\Delta z} \frac{1}{2\Delta^2} (\Psi_{i-1}^{n+1/2} - 2\Psi_i^{n+1/2} + \Psi_{i+1}^{n+1/2}) &= \Psi_i^n - j\beta_{\Delta z} \frac{1}{2\Delta_i^2} (\Psi_{i-1}^n - 2\Psi_i^n + \Psi_{i+1}^n) \\
\Psi_{i+1}^{n+1} &= \Psi_{i+1/2}^{n+1/2} \exp \left[ 2j \left| \Psi_i^{n+1/2} \right|^2 + \alpha \right] \Delta_i
\end{align*}
\]

(4)

We have chosen the Crank-Nicholson scheme among various FDM schemes for its good characteristics: it is second order accurate in time, first order accurate in space and it is unconditionally numerically stable. On the other hand, it is an implicit scheme and in each step the tridiagonal matrix has to be solved.

When modeling with SSFM, first we have to apply the Fourier transform on the equation (we used the fast Fourier transform – FFT). The equation in the spectral domain has an analytic solution within the segment \(\Delta z\) and we have just to apply the inverse FFT to transform the pulse envelope to have it back in the time domain. In the whole process, described by
\[
\begin{align*}
FFT \rightarrow \Psi_{i+1/2}^{n+1/2} &= \Psi_{i+1/2}^{n+1/2} \exp \left( j\beta_{\Delta z} \omega^2 \Delta_i \right) \rightarrow FFT^{-1} \\
\Psi_{i+1}^{n+1} &= \Psi_{i+1/2}^{n+1/2} \exp \left[ 2j \left| \Psi_i^{n+1/2} \right|^2 + \alpha \right] \Delta_i
\end{align*}
\]

(5)

the most of the computer time is needed for performing FFT and inverse FFT. When we compared both methods, FDM gave us better results concerning accuracy and time needed for computing than SSFT method and we used it for further simulations of the complete optical link.

We developed also models of an optical amplifier and optical filters to be able to simulate more complicated optical links. Among various optical amplifiers we have chosen Erbium-doped fiber amplifier (EDFA) and modeled it as an ideal amplifier that amplifies the input signal with the same gain \(G\) in the whole amplifiers bandwidth i.e. the gain is a rectangular function of frequency. Total noise power of the amplifier spontaneous emission (ASE) noise equals \(B_A S_{sp} = B_A (G-1) n_{sp} h\nu_0\) where \(S_{sp}\) represents power spectral density, \(B_A\) – the amplifier bandwidth, \(n_{sp}\) is the spontaneous emission factor or population-inversion factor and \(\nu_0\) amplifier’s central frequency defined as \(c/\lambda_0\). On the other hand, we know that white noise can be represented by spectral components with the same amplitude \(A_N\), therefore the total noise power can be written also as \(A_N^2 N_{SC}\) where
$N_{SC} = B_A / \Delta \nu, \quad \Delta \nu = 1 / WT$ is the number of spectral components and $WT$ equals the width of time window simulated. The noise in the time domain can now be modeled as a superposition of $N_{SC}$ sinusoidal functions according to the relation

$$N(t) = \sum_{i=1}^{N_{SC}/2} A_x e^{i(2\pi \Delta \nu t + \phi)}$$

(6)

where $\phi$ is a random phase and $\Delta \nu$ is defined above. The filters are modeled by the use of their transfer function in frequency domain and are thus the only component which is modeled in spectral domain. In optical filter modeling we used first the Fourier transform of the signal, then applied the chosen filter’s transfer function, and finally obtained the signal back in the time domain by applying the inverse Fourier transformation.

**Simulation Results**

In a modeling process we compared the accuracy of FDM and SSFM approaches described above as well as the time needed for simulations of both methods. The comparison was done on some canonical cases (Gaussian and soliton pulses), which have an analytical solution when propagating along the lossless fiber. As the measure of accuracy we have taken the autocorrelation function of analytic solution and simulation results ($1 - |AKC|$ and arg($AKC$) are given in Table I). Table II contains the results of how much computational time we needed for these canonical case simulations. The finite-difference Crank-Nicholson scheme is superior to the SSFM model according to both criteria and we used it in further work on simulations of the whole link.

**Table I** Comparison of FDM and SSFT on the basis of autocorrelation function

<table>
<thead>
<tr>
<th>Example</th>
<th>FDM</th>
<th>SSFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss</td>
<td>3.840E-05</td>
<td>4.400E-05</td>
</tr>
<tr>
<td>1\textsuperscript{st} order soliton</td>
<td>1.130E-05</td>
<td>1.390E-04</td>
</tr>
<tr>
<td>2\textsuperscript{nd} order soliton</td>
<td>1.175E-05</td>
<td>5.780E-04</td>
</tr>
<tr>
<td>3\textsuperscript{rd} order soliton</td>
<td>1.340E-03</td>
<td>0.285 1.9</td>
</tr>
</tbody>
</table>

**Table II** Comparison of FDM and SSFT on the basis of time needed for simulation

<table>
<thead>
<tr>
<th>Example</th>
<th>FDM (min.)</th>
<th>SSFM (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss</td>
<td>0.009</td>
<td>0.010</td>
</tr>
<tr>
<td>1\textsuperscript{st} order soliton</td>
<td>0.654</td>
<td>1.068</td>
</tr>
<tr>
<td>2\textsuperscript{nd} order soliton</td>
<td>0.536</td>
<td>0.950</td>
</tr>
<tr>
<td>3\textsuperscript{rd} order soliton</td>
<td>13.396</td>
<td>20.085</td>
</tr>
</tbody>
</table>

When one uses the finite differences, special attention is always given to initial values and boundary conditions. In FiberProp, the program we developed, one can choose between various initial pulse shapes in sequences of up to 16 bits. For the canonical test cases we used zero boundary conditions which are applied simply by setting the value zero to the points at each end of computational window.

For the more complicated cases of pulse sequences propagation along the optical links consisting of real fiber (with losses), erbium-doped fiber amplifiers and optical filters i.e. as soon as some dispersive waves exist, one has to use the absorbing boundary conditions (ABC) or transparent boundary conditions (TBC). The ABC describe the situation in which all the energy that comes to end of computational window is absorbed by adding extra $N/2$ grid points at each side of the computational window and assign a small absorption coefficient to them. In this way, the wave is slowly attenuated after it crosses the boundary. In implementing the TBC, on the other hand, one derives the relation between the wave and its derivative in the left or right boundary conditions.
point (different signs). The assumption that in the left (or right) boundary point exists only the portion of the wave propagating transversally to the left (or right) is used here. On this way, no reflection back into the computational window is obtained.

In Fig.1, the transmission of pseudorandom bit sequence 01111100 along the transmission line with parameters given in Table III is shown for two different boundary conditions – ABC and TBC. By comparing ABC and TBC simulations we concluded that less computational time is needed for the implementation of TBC. Fig.2. shows us the contour map for ABC simulation (additional points with small absorption coefficient are also drawn).

<table>
<thead>
<tr>
<th>Fiber parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ) [dB/km]</td>
<td>0.22</td>
</tr>
<tr>
<td>( \beta'' ) [ps²/km]</td>
<td>-0.243</td>
</tr>
<tr>
<td>( \gamma ) [W⁻¹·km⁻¹]</td>
<td>1.475</td>
</tr>
<tr>
<td>( L_0 ) [km]</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input pulse – sech shaped sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_0 ) [ps]</td>
</tr>
<tr>
<td>( N )</td>
</tr>
<tr>
<td>( q_0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EDFAs parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G ) [dB]</td>
</tr>
<tr>
<td>( \lambda_0 ) [nm]</td>
</tr>
<tr>
<td>( \Delta \lambda ) [nm]</td>
</tr>
</tbody>
</table>

Table III parameters of transmission line and input sequence

CONCLUSIONS

In the paper we describe the FDM and SSFM models of NLSE. FDM is found to be more accurate and also faster method (needs less computational time). The optical amplifier and filter models are also listed. Special attention in simulating the whole optical link is given to various boundary conditions used. ZBC is found to be applicable just in simplest transmission cases, while both TBC and ABC can be used for more demanding optical link simulations. In the paper we give an illustrative simulation of optical link and show on that example that using TBC results in faster simulations.

REFERENCES