Metrological and Transient Analysis of Combined Instrument Transformer Coupled with FEM-3D Magnetic Field Study

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Abstract

In the paper a 20 kV combined current-voltage instrument transformer (CCVIT) metrological and transient characteristics will be calculated by using numerical methods. The CCVIT is a non-linear electromagnetic system consisting of two magnetic measurement cores (voltage VT and current CT measurement core) with mutual magnetic influence and two electrical winding systems which increase the voltage, current and phase displacement errors. The non-linear iterative calculation of the magnetic field distribution in the three-dimensional domain is achieved by using the finite element method and an original and universal program package FEM-3D. The initial design of CCVIT is made according to the analytical transformer theory. The FEM-3D results are basis for the further genetic algorithm (GA) optimal design. The objective function in the optimization process is the minimum of the CCVIT steady state metrological parameters. The optimization GA coupled with FEM-3D methodology given in the paper derive positive design results from metrological aspect. After steady state metrological analysis is done, transient CCVIT characteristics are derived by coupling with the FEM-3D magnetic field study.

Introduction

The instrument transformers as part of the power measurement systems require high accuracy of transformed voltages and currents, [1].

The 20 kV combined current-voltage instrument transformer (CCVIT) is a non-linear electromagnetic system consisting of two magnetic measurement cores (voltage VT and current CT measurement core) with mutual magnetic influence and two electrical winding systems which increase the voltage, current and phase displacement errors, [2]. The analytical transformer theory analysis can be applied only on simple electromagnetic structures which is not the case with the CCVIT given in Figure 1. The numerical methods are indispensable. For the purposes of the three-dimensional magnetic field study by using the finite element method the 3D domain of the CCVIT is divided into 19 cross-sectional layers along the z-axis as shown in Figure 1: primary VT winding (1. and 10. layer), secondary VT winding (3., 8.), VT magnetic core (5., 6.), primary CT winding (12., 19.), secondary CT winding (14., 17.), CT magnetic core (15., 16.), isolation (2., 4., 7., 9., 11., 13., 18.). The initial design of the CCVIT is made according to the analytical transformer theory.

The magnetic field study in the complex three-dimensional domain of the CCVIT begins with the system of Poisson’s non-linear partial differential equations as follows:

$$\frac{\partial}{\partial x} \left( \frac{r}{v(B) \frac{\partial A}{\partial x}} \right) + \frac{\partial}{\partial y} \left( \frac{r}{v(B) \frac{\partial A}{\partial y}} \right) + \frac{\partial}{\partial z} \left( \frac{r}{v(B) \frac{\partial A}{\partial z}} \right) = -j(x,y,z)$$  (1)
where $\mathbf{A}$ is the magnetic vector potential as an auxiliary quantity, $\mathbf{B}$ is the magnetic flux density, $\nu$ is the magnetic reluctivity and $j_v$ is the volume current density. The CCVIT is a closed and bounded electromagnetic system with prescribed boundary conditions.

**FEM-3D Calculation of Leakage Reactances Characteristics**

The non-linear iterative calculation of the magnetic field distribution in the three-dimensional domain is achieved by using the finite element method and an original and universal program package FEM-3D developed at the Faculty of Electrical Engineering-Skopje, [3]. The input primary voltage (for the VT core) and the input current (for the CT core) are changed from plug out regime to 120% of their rated values (VT rated ratio: $20000 \sqrt{3} \div 100 \sqrt{3}$ and CT rated ratio: 100 A : 5 A). FEM-3D enables exact calculation of the leakage reactances of the four CCVIT windings given in Fig. 2-5.

**Genetic Algorithm Optimal Design**

The FEM-3D results are basis for the further genetic algorithm (GA) optimal design. The objective function in the optimization process is the minimum of the CCVIT steady state metrological parameters, the mutually coupled voltage error $p_v$ and the current error $p_c$:

$$f_{\text{opt}} = \frac{1}{1 + |p_c|} + \frac{1}{1 + |p_v|} + \frac{1}{1 + |p_v0.25|} + \frac{1}{1 + |p_v + p_v0.25|} \quad (2)$$

The rated regime for the both measurement cores has been selected for the metrological optimal design of the CCVIT. In the CCVIT mathematical optimization model all the quantities which affect the objective function are made to be dependent on 11 input optimization variables given in Table 1. The optimization is done by the original program developed at the Faculty of Electrical Engineering in Skopje, [4]. The optimal design CCVIT project is adjusted for practical realization of a CCVIT prototype by adopting standardized geometrical and building factors which leads to the optimized design project. The optimization GA coupled with FEM-3D methodology given in the paper derive positive design results from metrological aspect.
Table 1. Input design variables in the genetic algorithm optimization process

<table>
<thead>
<tr>
<th>design variable</th>
<th>mapping range</th>
<th>initial</th>
<th>optimal</th>
<th>optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of turns of primary VT winding</td>
<td>23584 - 24000</td>
<td>24000</td>
<td>23655</td>
<td>23655</td>
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<tr>
<td>primary VT winding current density [A/mm²]</td>
<td>1.5 - 3.0</td>
<td>2.04</td>
<td>1.509</td>
<td>1.617</td>
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<tr>
<td>secondary VT winding current density [A/mm²]</td>
<td>1.5 - 3.0</td>
<td>2.61</td>
<td>2.000</td>
<td>1.677</td>
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<tr>
<td>VT magnetic core outside length [mm]</td>
<td>183 - 221</td>
<td>185</td>
<td>191.187</td>
<td>191</td>
</tr>
<tr>
<td>VT magnetic core depth [mm]</td>
<td>49 - 54</td>
<td>50</td>
<td>53.44</td>
<td>53</td>
</tr>
<tr>
<td>number of turns of secondary CT winding</td>
<td>115 - 125</td>
<td>120</td>
<td>119</td>
<td>119</td>
</tr>
<tr>
<td>primary CT winding current density [A/mm²]</td>
<td>1.0 - 1.6</td>
<td>1.36</td>
<td>1.0198</td>
<td>1.587</td>
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<tr>
<td>secondary CT winding current density [A/mm²]</td>
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<td>2.55</td>
<td>2.495</td>
<td>2.548</td>
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<tr>
<td>primary CT winding copper width [mm]</td>
<td>9.0 - 11.0</td>
<td>10.5</td>
<td>9.00</td>
<td>9.00</td>
</tr>
<tr>
<td>CT magnetic core outside length [mm]</td>
<td>136 - 162</td>
<td>142</td>
<td>156.31</td>
<td>148</td>
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<tr>
<td>CT magnetic core depth [mm]</td>
<td>15 - 60</td>
<td>25</td>
<td>21.15</td>
<td>17.00</td>
</tr>
</tbody>
</table>

**CCVIT Metrological Characteristics**

The metrological CCVIT characteristics of the initial, optimal and optimized design project are compared in Figures 6-9.

- Figure 6. VT voltage error characteristics via the input VT voltage, CT in rated regime
- Figure 7. VT phase displacement error characteristics via the input VT voltage, CT in rated regime
- Figure 8. CT current error characteristics via the input CT current, VT in rated regime
- Figure 9. CT phase displacement error characteristics via the input CT current, VT in rated regime

The designed CCVIT from the initial accuracy class 3 of the voltage transformation core and accuracy class 1 of the current transformation core of the analytical design has been optimized in order to be achieved higher accuracy class: 1 of the VT core and 0.1 of the CT core of the optimal CCVIT and 1 of the VT core and 0.5 of the CT core of the optimized CCVIT.

**CCVIT Transient Characteristics**

After the steady state metrological analysis is done, transient CCVIT characteristics are derived by coupling with the FEM-3D magnetic field study. The leakage reactances characteristics are input data into the non-linear mathematical model of the CCVIT. The CCVIT transient analysis characteristics of the plug in primary currents of the both measurement cores (VT and CT) in the three design projects (analytical, optimal and optimized) are displayed in Figures 10 and 11. In the Figures 12 and 13, the
maximal values of the secondary current of the both cores in the three design project are compared. The complex non-linear analysis has been done by using the program package Matlab/Simulink for rated loads of the both measurement cores and rated frequency of 50 Hz. The displayed transient characteristics are for the most critical moment for the CCVIT, 5 ms (1/4 of the input signal period of 20 ms), when the maximum of the current is achieved.

![Figure 10. Transient characteristics of the primary VT plug in current maximum](image1)

![Figure 11. Transient characteristics of the primary CT plug in current maximum](image2)

![Figure 12. Transient characteristics of the secondary VT current maximum](image3)

![Figure 13. Transient characteristics of the secondary CT current maximum](image4)

Conclusion

In the paper the original three-dimensional finite element magnetic field analysis has been coupled with genetic algorithm for metrological optimal design of the combined current-voltage instrument transformer. The optimal design has derived improved instrument transformer from metrological as well as from transient phenomena aspect. A complex steady state metrological and transient analysis of the CCVIT performance has been accomplished. The optimal CCVIT has the highest accuracy class and it is with the lowest plug in current characteristics compared to the analytical and optimized CCVIT. The methodology presented in this paper is universal and can be applied for the analysis of other non-linear electromagnetic systems.

REFERENCES